

# Application Note

# 8

## *Digital FIR Interpolator & Analog Lowpass*

### **Highlights**

*Multirate FIR Design*  
*Cascade Analog Lowpass*  
*Circuit Optimization*  
*Analog Filter Correction*  
*Sin(x)/x Correction*

### **■ Design Objective**

1:4 FIR Interpolation Filter  
Fs: Input = 250kHz, Output=1MHz  
110kHz Bandwidth, Linear Phase  
0.01dB Passband Flatness  
100dB Stopband Attenuation

This design will provide an example of how to integrate both the analog and digital simulation features of the program. A high performance interpolation filter will be produced. These filters are often used for DAC output reconstruction, and also in other situations where narrow-band low frequency filtering is required.

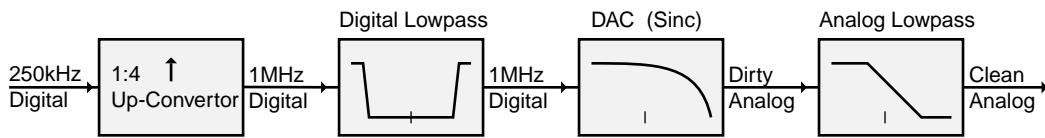
This example could arise from an instrumentation system, or general purpose data acquisition system, where the main internal processing data rate was 250kHz. The time domain data must now be converted back into analog via DAC.

If the 250kHz data was fed directly into the DAC, a highly selective analog elliptic filter would be required for reconstruction to remove the image band above the Nyquist frequency of 125kHz. Using multirate techniques, an oversampling reconstruction filter can be produced with greatly improved performance.

Interpolation is the process of upshifting the data (sampling) rate to a higher frequency. This then enables digital filtering based on the higher frequency. By increasing the data rate digital filtering can be applied directly at the main passband edge. The analog filter is then used to remove the much higher image products, allowing use of a wider transition band. In contrast to the previous approach, this method requires DSP and the use of a DAC with 4X higher speed capability.

### ■ Basic Filter System Description

A 1:4 total interpolation ratio will be used. Therefore the output DAC will need to support 1MHz data rates. The diagram below illustrates the mixed signal flow of the filter:



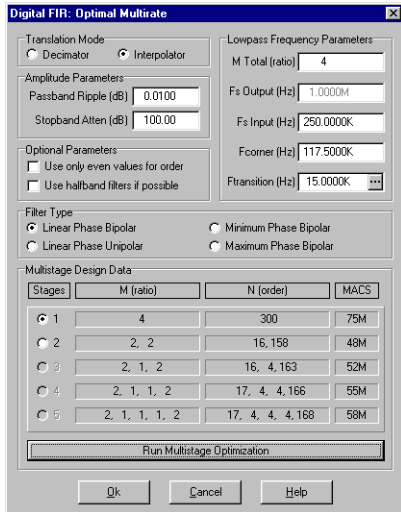
The markers in each of the pictograms above indicate the Nyquist frequency relative to the 1MHz output sample rate ( $F_n=500\text{kHz}$ ). The Up-Converter increases the sampling rate by adding zero samples between the input samples. This is a simple DSP operation, and a 1:4 converter would merely add three zero samples between every original sample.

The digital Lowpass filter provides the critical cutoff near 125kHz. The image band then lies above 875kHz as shown here. The DAC contains a sample & hold which produces a  $\sin(x)/x$  (sinc) transfer function at 1MHz. After the DAC converts the signal to analog, the analog Lowpass filter then removes the high frequency image band.

In actual implementation the up-converter is effectively built into the digital Lowpass filter by algorithm. Since only one out of every four input samples contains nonzero data, the output FIR computations can be constructed to skip 3 out of 4 coefficients for every output sample.

The transition bandwidth for the analog Lowpass filter is now between 125kHz to 875kHz. This provides a much wider region to reach the high stopband attenuation.

For this design a linear frequency range from 0Hz to 1MHz will be used, so that all of the upper image bands will be visible. The Data Curves will be generated with 500 point resolution.



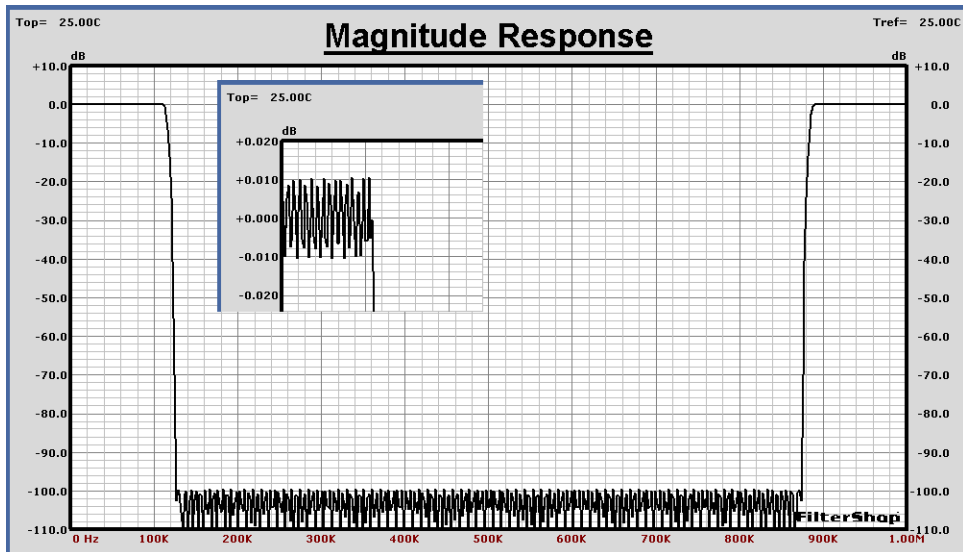
■ **Multirate FIR Filters**

We now wish to introduce one of the Target system FIR filter design dialogs: *Digital-FIR: Optimal Multirate*. This dialog greatly simplifies the design of FIR interpolators and decimators.

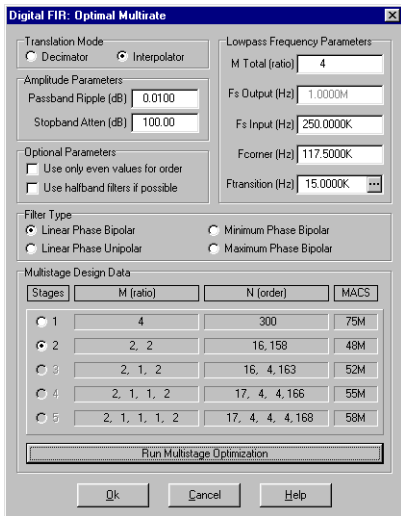
The parameters in the dialog are setup as shown here on the left. The Interpolator selection is chosen, 0.01dB passband ripple, 100dB stopband attenuation, the M ratio set to 4, and the input sampling rate set to 250kHz.

In choosing the corner and transition frequency, we click the small [...] button and enter the edge frequencies of 110kHz and 125kHz. If we wish to have clean transition bands, the full attenuation must be reached at 125kHz. The resulting corner and transition frequencies are 117.5kHz and 15kHz respectively.

Note that the *Multistage Design Data* table is currently empty. Click the *Run Multistage Optimization* button. The table is now filled with various values. The default selected is #2, however for now we shall select #1 and then click *Ok*. The magnitude response shown below is produced.



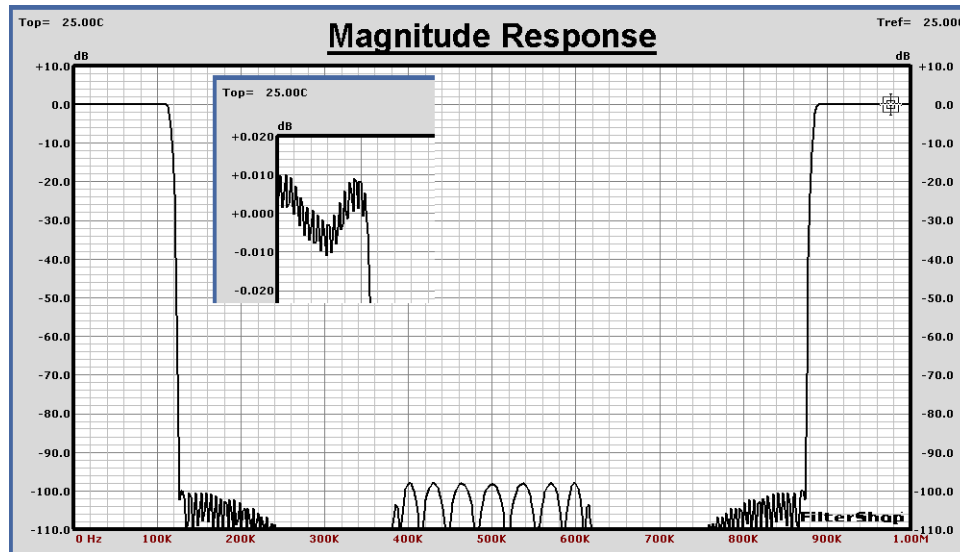
As described the passband covers 0Hz to 110kHz, with the image band between 875kHz and 1MHz. The stopband shows the desired 100dB attenuation, and the passband ripple insert shows 0.01dB. The analog Lowpass will need to remove the upper image band.



Now let's look at the multirate dialog again. The parameters are the same as before, so just click the *Run Multistage Optimization* button to fill the table.

Note there are two *Stages* options available: 1, and 2. We just used the 1 stage option which created a single FIR filter using the interpolation ratio of 1:4. However, we could also perform this same function in two stages each using an interpolation ratio of 1:2. This design is represented by the 2nd option.

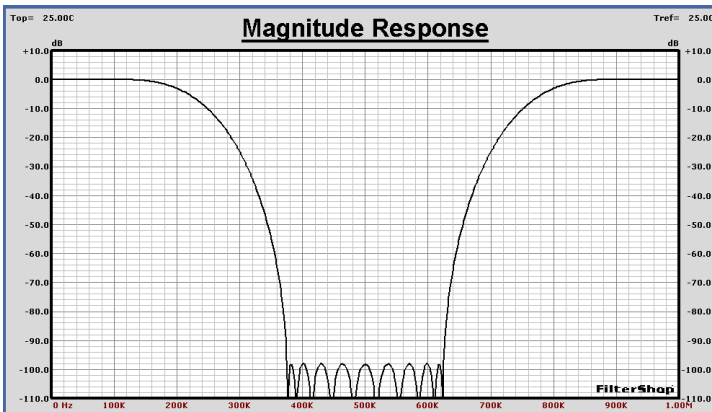
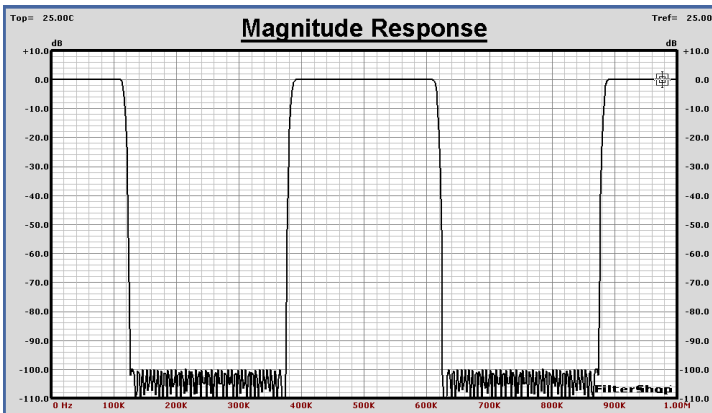
The *N (Order)* column shows the required order for each stage, and here it is 16 and 158. The last column shows the MACS value (number of multiply accumulates per second). Note that using two stages reduces the computational burden by 1/3. Choosing this two stage option produces the magnitude response below.



The passband/stopband regions cover the same frequencies as before, but the stopband now shows the combined response of two different FIR filters.

The FIR filters created by the Multirate dialog in the Target Parameters are shown below. Each operates at a different sampling rate. If each of the two TFBs are enabled separately, the magnitude response shown below is produced.

Target Parameters													
TFB	Mag	Phs	Eqn Gp	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz	
1	Inv	Rev	M1	FIR	0.0000	1.0000M			16	H(0000) = -1.2030m			
2	Inv	Rev	M1	FIR	0.0000	500.0000K			158	H(0000) = -32.7129u			



Both of the two FIR filters are providing stopband attenuation, but indifferent frequency regions. The best choice of FIR order for each of the two stages was determined by the Multirate dialog.

By spreading the interpolation ratio across multiple stages, greater efficiency can be obtained. In this case the savings was only 35%, but in other cases it can be many times larger.

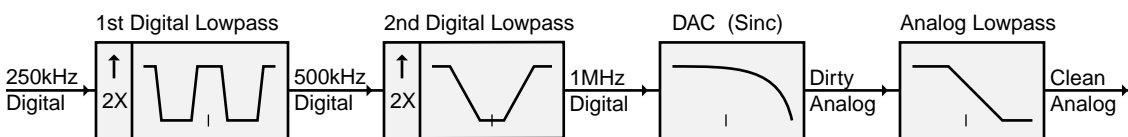
Note that the 2nd FIR stage does not quite meet the requested 100dB stopband attenuation. This is a common occurrence when designing optimal filters.

The FIR order prediction for optimal filters is based on an *estimate*. The actual response is unknown until the final result is obtained by numerical solution.

To correct this we could simply rerun the Multirate design dialog and request slightly more attenuation. However, for this example we will also need to include other response corrections.

The Multirate dialog provides a quick means of determining the best possible orders for different stage combinations. It also calculates the proper edge frequencies needed for each FIR stage.

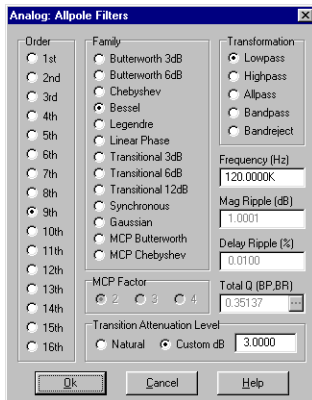
In this case, the two stage design will be used. We now know the general order and layout requirements of both FIR stages. The revised two-stage filter system model is shown below.



The 1st FIR stage increases the sampling rate from 250kHz to 500kHz. This is the high order stage (N=158), and provides the sharp cutoff from 110kHz-125kHz. It will also be used to correct the other passband response defects.

Because the sampling frequency of the 1st FIR is 500kHz, the 2nd FIR stage is used to remove the middle image band. This stage has very wide transition, and therefore the order is very low (N=16).

If the design had only called for a basic flat response interpolation filter, the FIR filters produced by the Multirate dialog would be final. However, in this example we need to correct the droop of the analog Lowpass and DAC sinc functions. For this we must recreate the FIR stages individually, where additional transfer functions can be built into the FIR response.

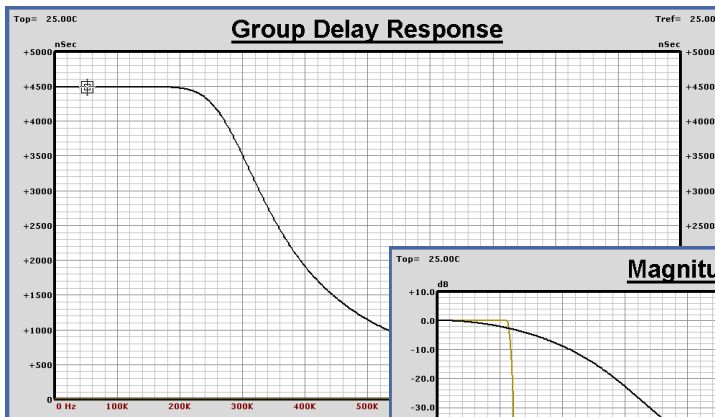


**Analog Filter Target**

We need 100dB of attenuation from the analog Lowpass filter at 875kHz. Since the overall design calls for linear phase response, the type of analog filter family is limited. The appropriate choices would be Bessel, Linear Phase, or Gaussian.

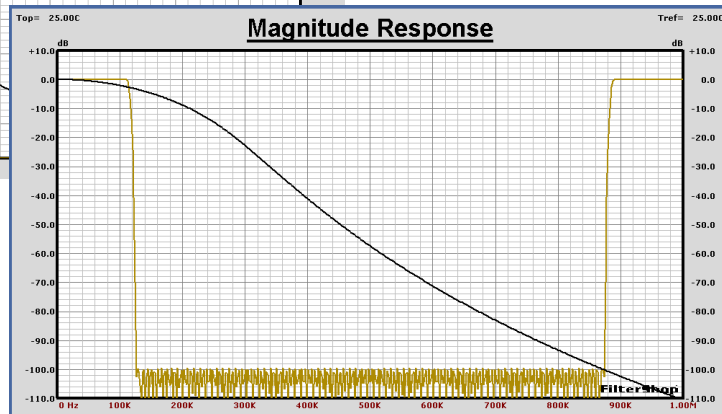
Using the previous FIR filter response as a visual guide, several different Allpole filters were tried. The Linear Phase family provides the best attenuation rate while maintaining phase linearity. However the frequency range is relatively high and it is best to keep the Q requirements to a minimum. A 9th order Bessel filter using a corner frequency of 120kHz (relative to 3dB transition) was found to meet the required attenuation at 875kHz.

The magnitude and group delay response of the analog filter target is shown below. The filter provides pure linear phase across the entire 110kHz bandwidth, and achieves 100dB attenuation at 875kHz.



However the magnitude response of the filter in the 110kHz pass-band is not flat. The response is -3dB at the high frequency edge.

We will correct the magnitude droop in the 1st FIR Lowpass.



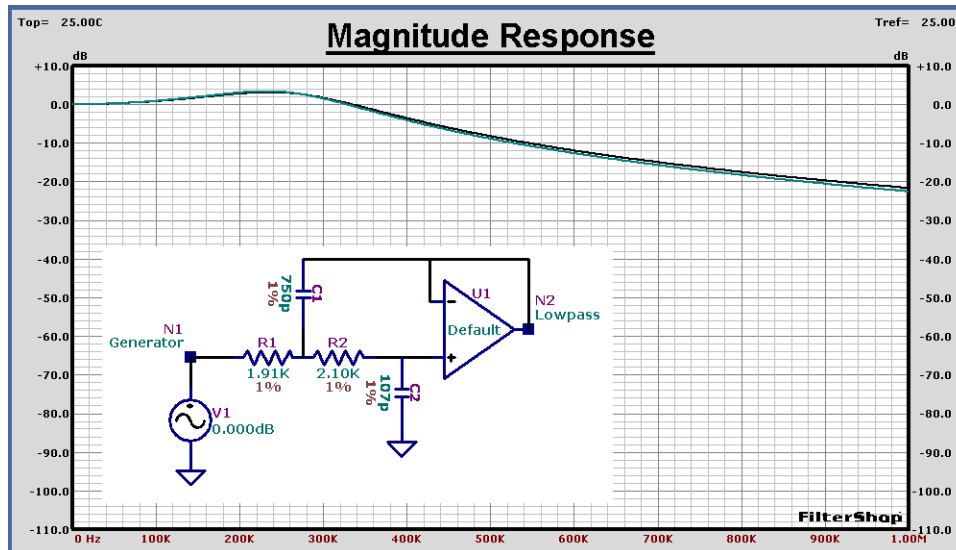
Target Parameters												
TFB	Mag	Phs	Eqn Gp	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Inv	Rev	M1	LP2	0.0000	279.1488K	1.3219			704.9180K	1.0000	
2	Inv	Rev	M1	LP2	0.0000	250.0688K	0.7506			861.0955K	1.0000	
3	Inv	Rev	M1	LP2	0.0000	234.1372K	0.5894			1.5100M	1.0000	
4	Inv	Rev	M1	LP2	0.0000	225.7878K	0.5197			1.8481M	1.0000	
5	Inv	Rev	M1	LP1	0.0000	223.1670K	0.5751			700.0000K	1.0000	

### ■ Analog Filter Circuit

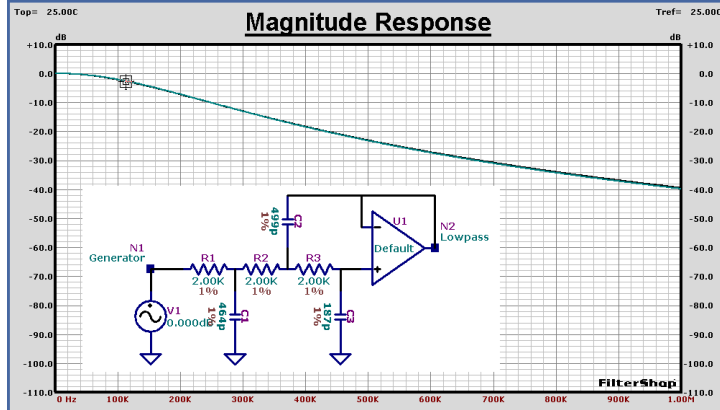
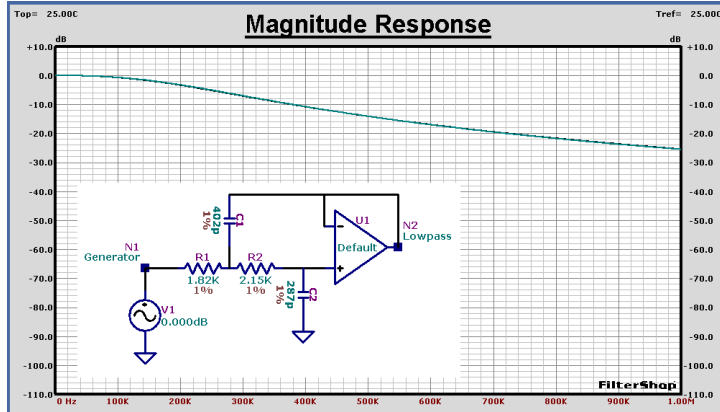
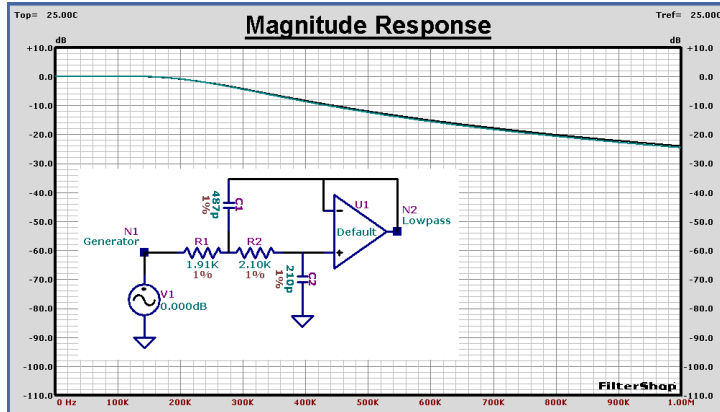
The final Target Parameters for the 9th order analog filter are shown above. Four unity gain Sallen-Key sections will be used to realize the Lowpass circuit: three 2nd order sections, and one 3rd order section.

A design file is created for each section, by saving/renaming the current design under different names. To begin the first section design, the TFBs 2-5 are disabled, and the LP2\_SK1 synthesis circuit loaded. After Synthesis is run, the following circuit and response is produced.

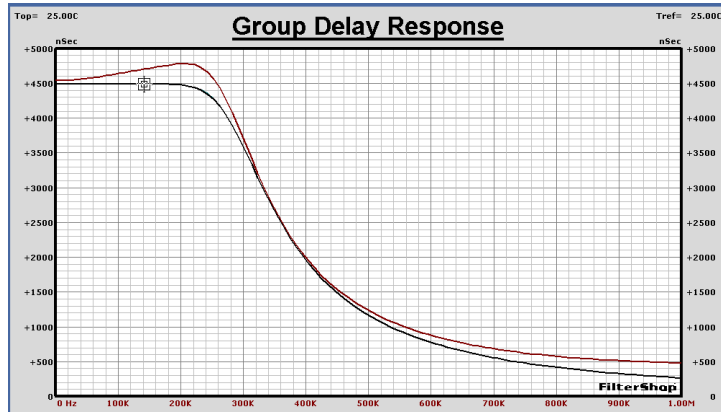
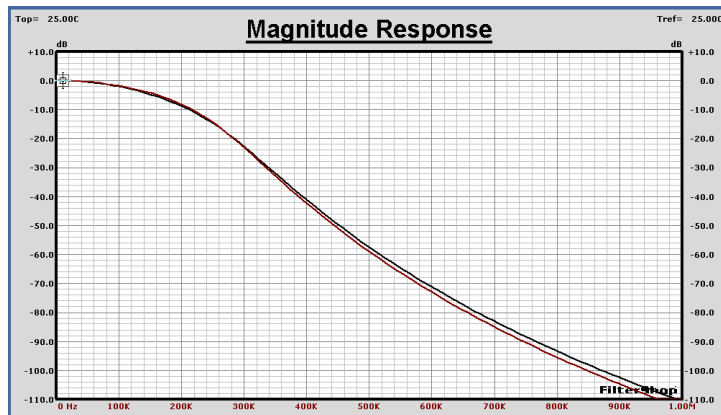
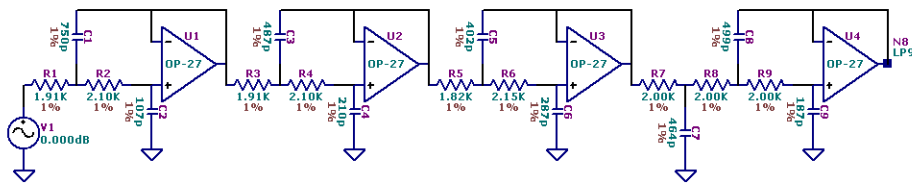
There is a slight difference between the target and the circuit output. This is due to the finite gain bandwidth of the opamp, and results in Q enhancement. This will be corrected later. The remaining three sections are shown on the following page, with the magnitude response of the circuit and ideal target.







Another design file is saved/renamed and all four sections are then imported. After deleting the unnecessary components, and connecting the sections, the following complete 9th order circuit is produced as shown below. The magnitude and delay response is also shown along with the ideal target.



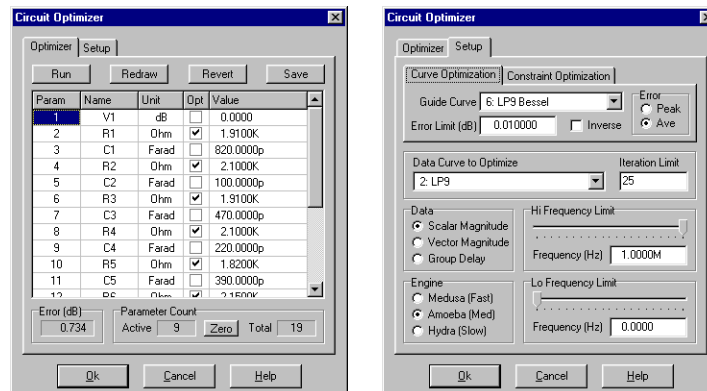
The magnitude response of the circuit differs slightly from that of the ideal target. However this is relatively unimportant since the magnitude response will be corrected later using the FIR Lowpass. The more serious problem concerns the group delay. It is no longer flat.

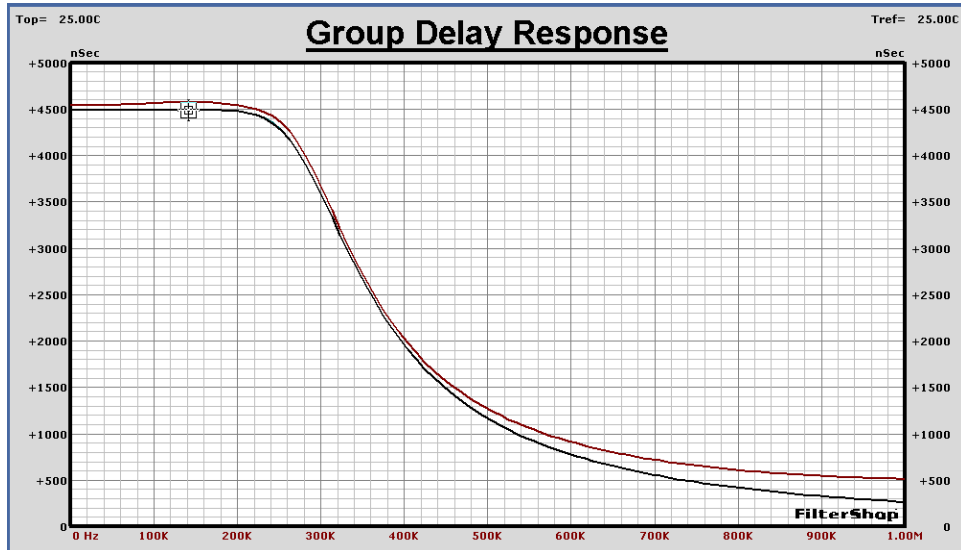
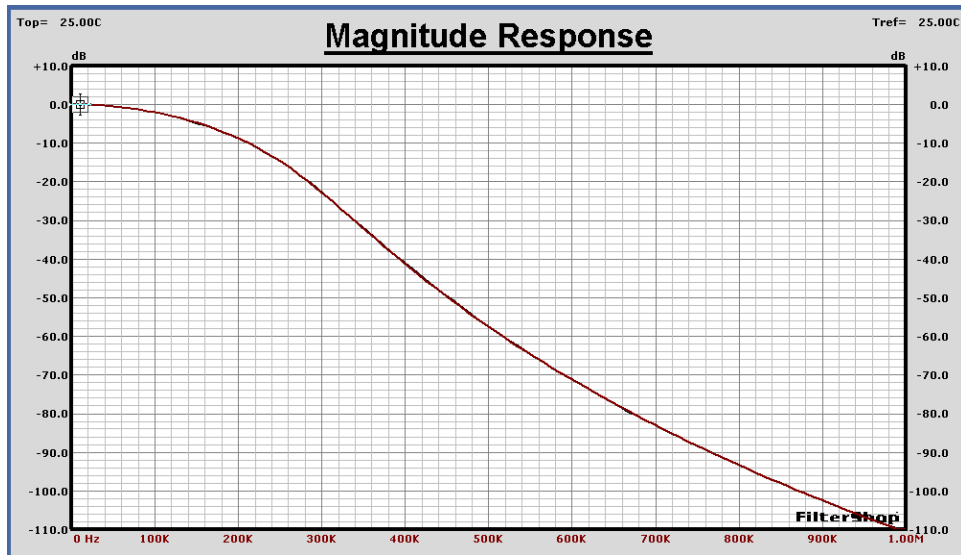
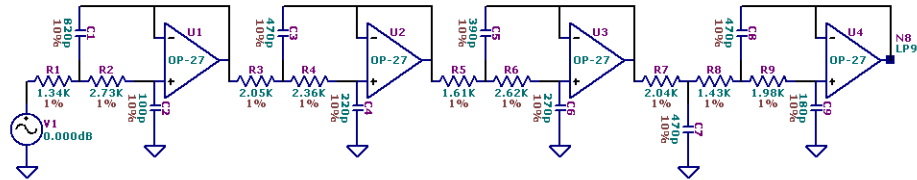
The circuit will be optimized to realign it with the target magnitude. But first the precision of the capacitors will be changed to 10% to round the current values to more common standards. Also, since the opamps will have a second order effect on the response of the circuit, it is important to analyze the circuit with a specific opamp. An OP-27 model will be assigned to all four.

The optimization frequency range will be the entire 0Hz to 1MHz, and only the nine resistors will be optimized. The optimizer setup is shown below.

When completed, the results on the following page are produced. Both the magnitude and delay response are greatly improved. The magnitude is essentially right on top of the target. However while the delay response has been substantially flattened, it remains at a higher level than the target. In fact the delay response of the circuit is higher than the target everywhere.

The extra delay is a result of the internal transfer functions of the opamps. At these higher frequencies (100kHz-1MHz), the finite bandwidth of the opamps produces noticeable delay.





We are not that concerned with the actual level of the delay, but that the delay remain flat in the 110kHz passband. Since the circuit wants to produce more delay, we must provide it with a new objective target curve that it can match.

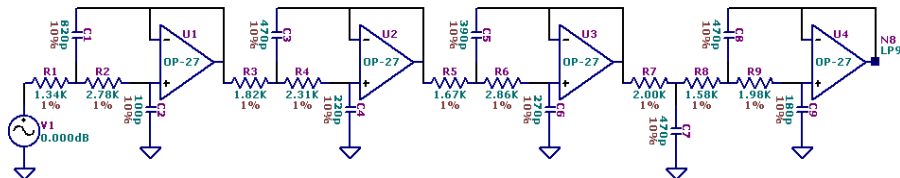
At 100kHz, there is about 85uS difference between the target and circuit. We can add this amount of delay to the target by multiplying a 1st order FIR TFB and set the  $F_s = 11.7\text{MHz}$ . The  $h_0$  coefficient of the FIR is zero, and the  $h_1$  coefficient is unity. Thus a simple delay of one sample period.

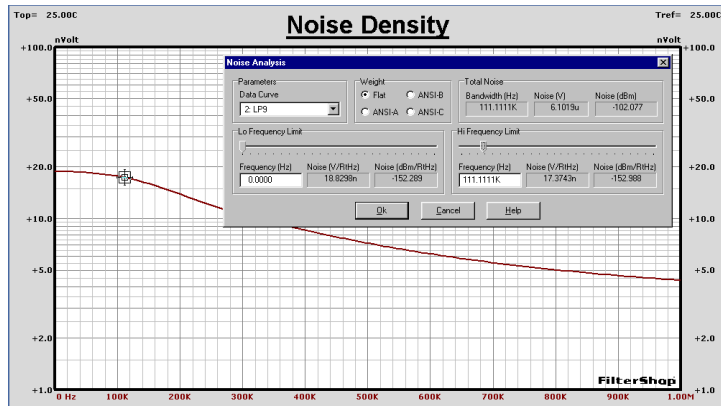
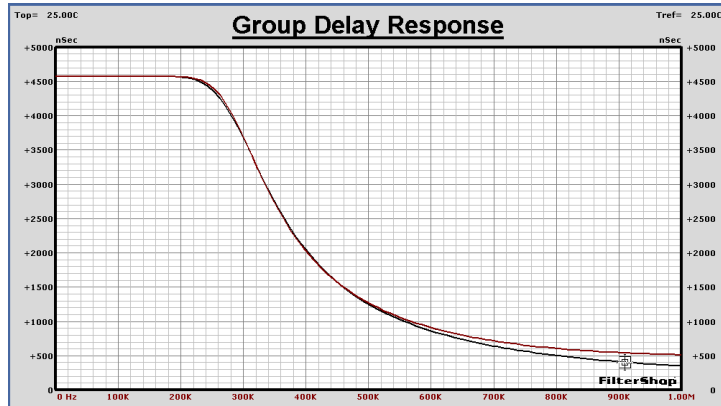
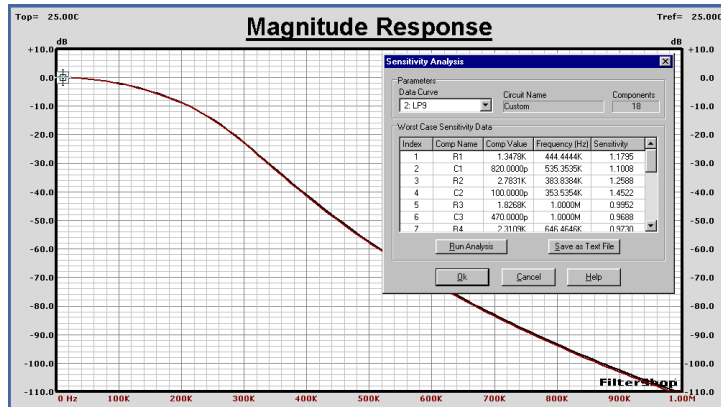
Target Parameters												
TFB	Mag	Phs	Eqn Gp	FilterType	Ao (dB)	Fp, Fz (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Invl	Rev	M1	LP2	0.0000	279.1488K		1.3219		704.8180K	1.0000	
2	Invl	Rev	M1	LP2	0.0000	250.0688K		0.7606		861.0955K	1.0000	
3	Invl	Rev	M1	LP2	0.0000	234.1372K		0.5894		1.5100M	1.0000	
4	Invl	Rev	M1	LP2	0.0000	225.7876K		0.5197		1.8481M	1.0000	
5	Invl	Rev	M1	LP1	0.0000	223.1670K		0.5751		700.0000K	1.0000	
6	Invl	Rev	M1	LEQ	0.0000	16.0000M		0.7071		1.3317K	1.0000	
7	Invl	Rev	M1	FIR	0.0000	11.7000M			1	H(0.000) = 0.0000		

The new target response is copied to another Guide Curve. The nine resistors of the circuit are now optimized again, but this time they are optimized to the *group delay* response rather than the magnitude. Also, the optimization frequency range is restricted from 0Hz to 200kHz. The only critical region is the passband, and this needs to be absolutely flat.

The final circuit is shown below. The magnitude and delay response is shown on the next page. While the magnitude has not been significantly affected, the delay is now very flat in the passband region. The sensitivity analysis gives excellent S values at less than 1.45 for all components.

The noise density of the final circuit is also shown on the following page. The total noise across the 110kHz passband is about 6uV or -102dBm. Based on a opamp clipping level of +20dBm, the SNR or dynamic range is about 122dB.



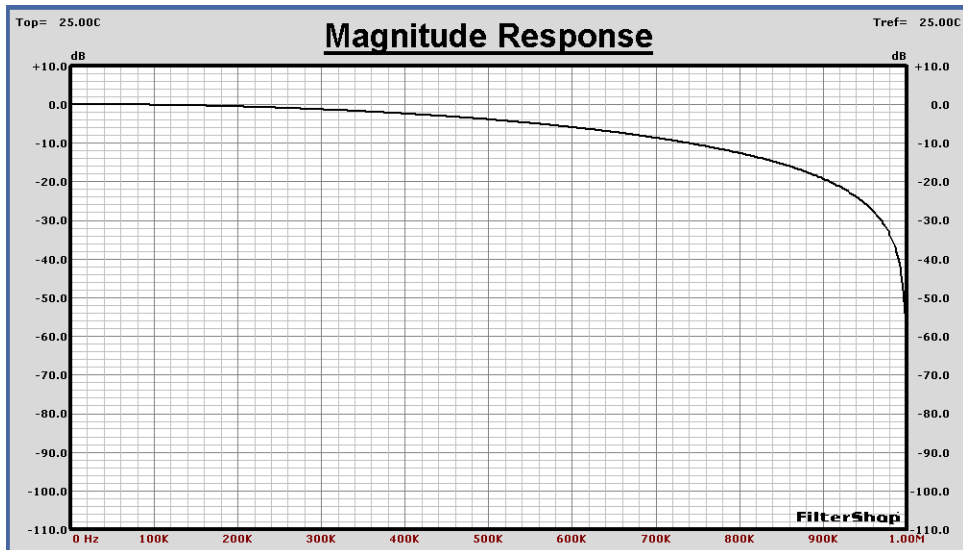


**■ DAC Sin(x)/x Transfer Function**

As mentioned previously we also wish to correct for a sinc response based on the 1MHz DAC frequency. This can be trivially created in the Target Parameters using a single SINC filter type TFB with  $F_s = 1\text{MHz}$ . This is shown below along with its magnitude response.

While the sinc response appears nearly flat in the 110kHz passband, the response actually has a droop of almost -0.2dB at 110kHz. If we are to meet the 0.01dB passband flatness specification, this must be included in the design.

Target Parameters												
TFB	Mag	Phs	Eqn Gp	FilterType	Ao (dB)	Fp, Fc (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Invl	Revl	M1	LP2	0.0000	279.1488K		1.3219		704.8180K	1.0000	
2	Invl	Revl	M1	LP2	0.0000	250.0688K		0.7606		861.0955K	1.0000	
3	Invl	Revl	M1	LP2	0.0000	234.1372K		0.5894		1.5100M	1.0000	
4	Invl	Revl	M1	LP2	0.0000	225.7878K		0.5197		1.8481M	1.0000	
5	Invl	Revl	M1	LP1	0.0000	223.1670K		0.5751		700.0000K	1.0000	
6	Invl	Revl	M1	LEQ	0.0000	15.0000M		0.7071		1.3317K	1.0000	
7	Invl	Revl	M1	FIR	0.0000	11.7000M			1	H(0.000) = 0.0000		
8	Invl	Revl	M1	FIR	0.0000	11.7000M				H(0.000) = 0.0000		
9	Invl	Revl	M1	Sinc	0.0000	1.0000M		0.7071		1.3317K	1.0000	



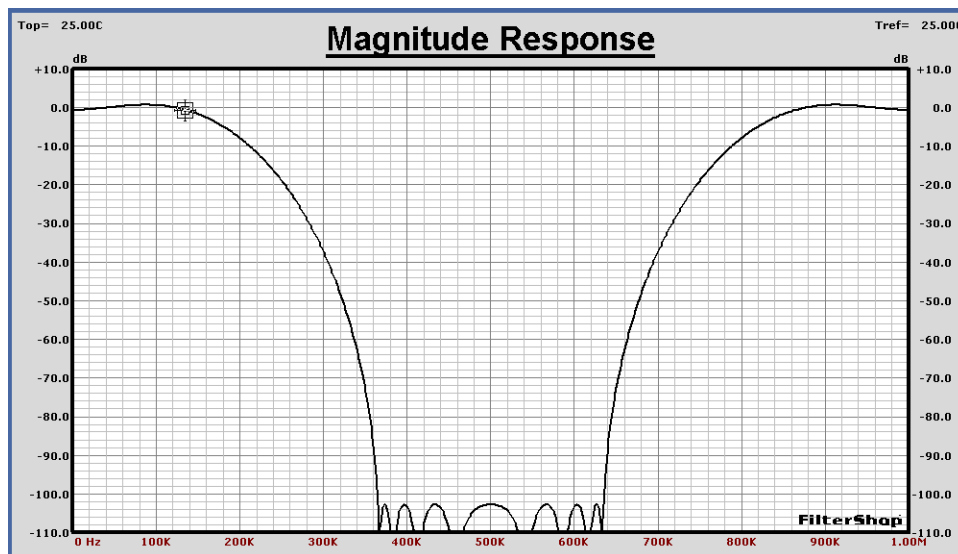
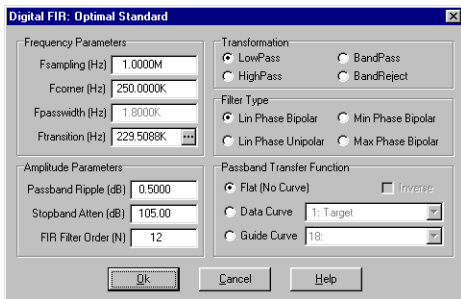
**■ 2nd Stage FIR Interpolator Design**

We are now ready to revisit the FIR interpolation filters. It was previously decided that a two stage interpolator would be used. Each stage interpolates by a 1:2 ratio. The preliminary analysis showed that one stage required an order of  $N = 16$  while the other required  $N = 158$ .

The low order 2nd FIR stage must be designed first, because it will produce passband defects which will be corrected in the 1st FIR stage.

Examining the previous two stage FIR response shows that this stage must have the stopband edge frequency at 375kHz. The actual passband ripple used by the Multirate dialog was 1/2 the requested 0.01dB total value, and this produced the order  $N=16$ . The flatness of this stage is unimportant, since it can be corrected in the other FIR stage. A ripple of 0.5dB will now be used.

For this single FIR filter, the *Digital-FIR: Optimal Standard* dialog is used. The sample rate for this stage is 1MHz, and edge frequencies of 125kHz and 375kHz are entered. To ensure that we have enough attenuation, a value of 105dB is entered. The order estimate is now reduced to 12. The magnitude response is shown below.

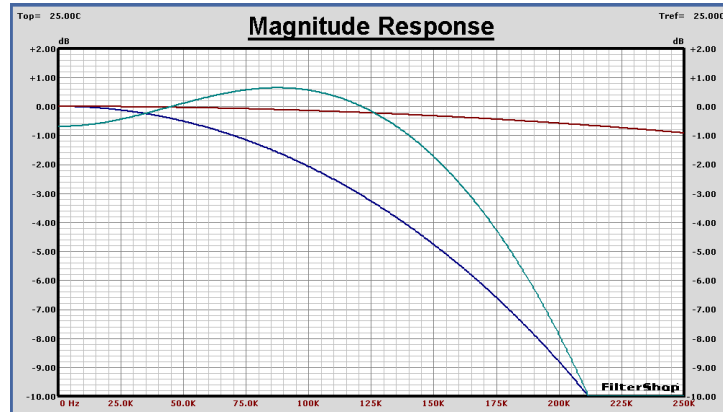
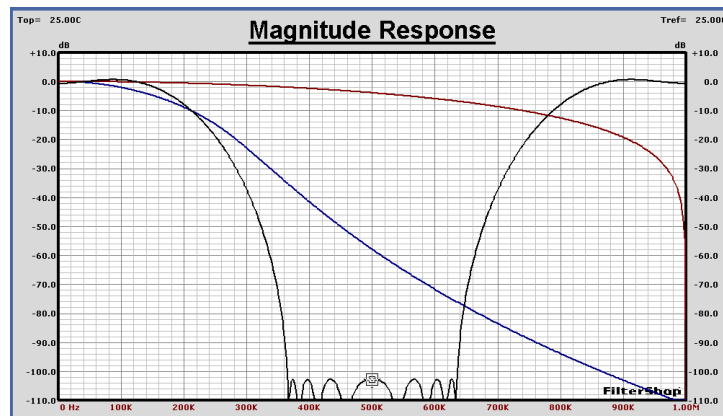




**Combined Response for Correction**

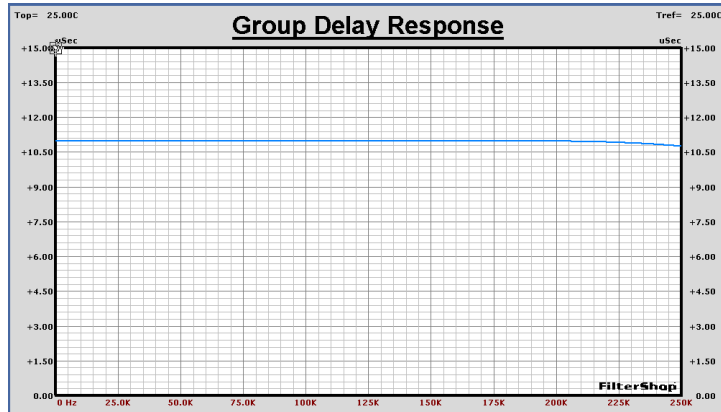
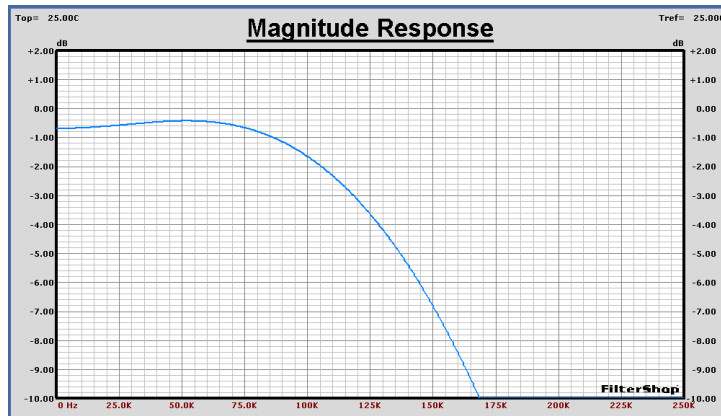
The magnitude response of the analog 9th order Lowpass, Sinc, and previous 12th order FIR filter are shown below. All three of these transfer functions cause defects in the passband region. By multiplying these together a single error response curve can be produced which will be used for correction.

However, the current frequency range extends to 1MHz and we are now only interested in the 110kHz passband, which uses roughly 10% of the total frequency data points in the curves. More resolution should be utilized for the passband region. The system frequency range is changed to 0Hz - 250kHz, and all three curves are recalculated. This is shown in the 2nd graph below.



The *Processing / Binary Math Operations* dialog is used to perform the curve multiplication. Two curves are first multiplied into one, and then that result is multiplied with the third curve. The magnitude and delay result is shown below.

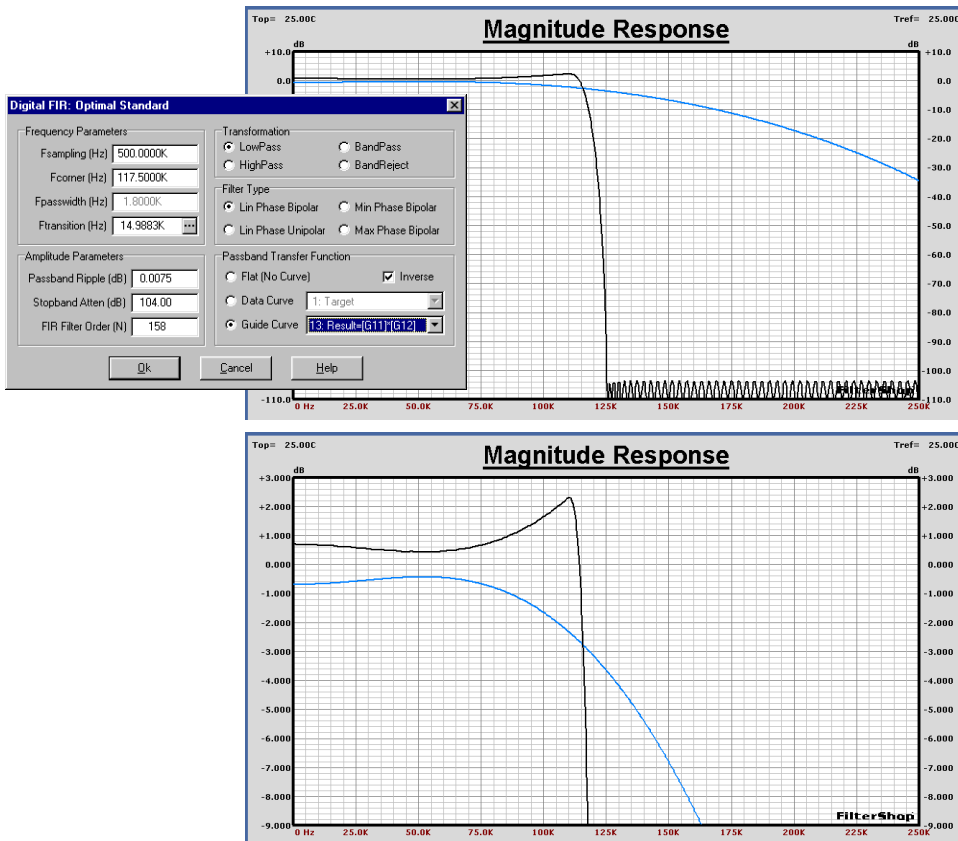
This single curve now contains all of the combined magnitude errors of the three transfer functions: analog Lowpass, Sinc, and the 2nd FIR/N=12 stage. By building the inverse of this curve into the passband response of the 1st high order FIR stage, the entire filter can be corrected perfectly.



■ 1st Stage FIR Interpolator Design

The 1st stage high order FIR filter corrects all of the previous magnitude errors, in addition to providing the precision cutoff at the 110kHz edge. Here we again use the *Digital-FIR: Optimal Standard* dialog.

This filter uses  $F_s = 500\text{kHz}$ , with edge frequencies of 110kHz and 125kHz. We will slightly understate the ripple as 0.0075dB and the attenuation as 104dB to ensure the solution meets the specifications. The final step is to select the Guide Curve containing the combined error response and enable *Inverse*. The estimated order is the same  $N = 158$ . The resulting FIR response is shown below. The complementary shape in the passband of the FIR filter is clearly visible.

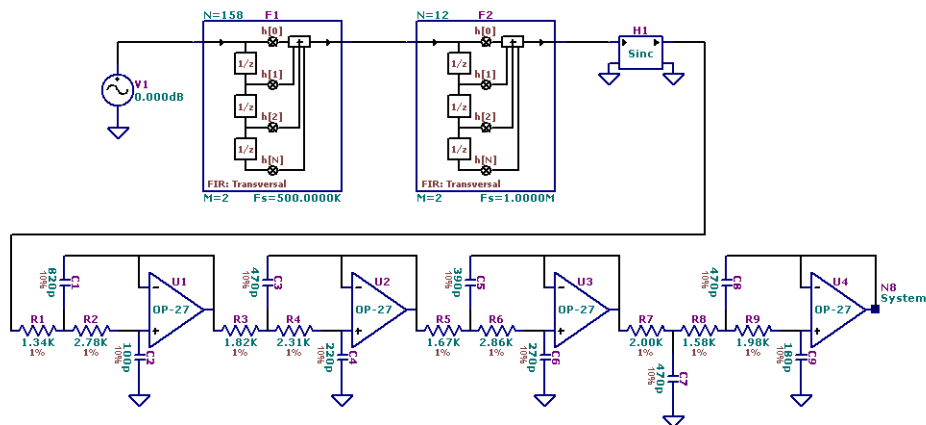


**Complete Filter Response**

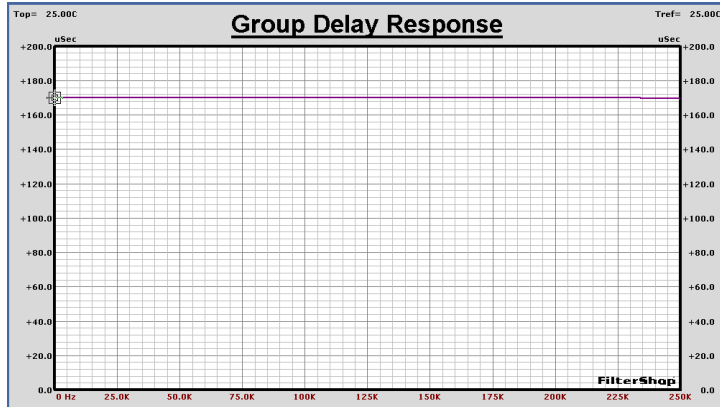
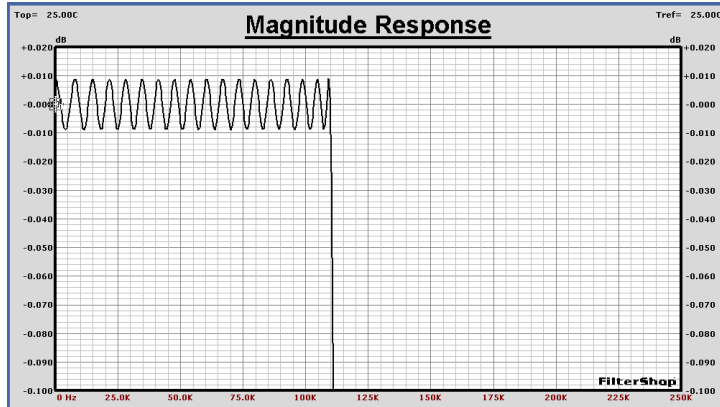
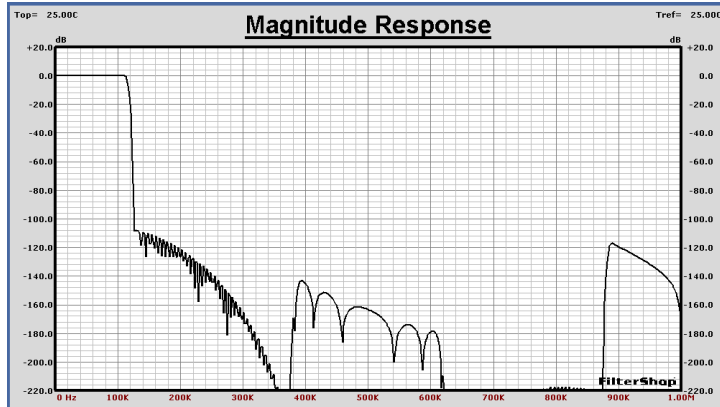
The entire interpolation filter design can now be verified. A circuit is created as shown below to provide the frequency domain simulation. Two FIR components are used for each of the two interpolation FIRs, and the target coefficient data is transferred into the components. An H component is used and setup as the Sinc function at 1MHz. The remaining 9th order analog Lowpass filter follows.

The three graphs on the following page display the results. The first graph shows the entire frequency range from 0Hz to 1MHz with 20dB/Div. All stopband regions meet and/or exceed the 100dB requirement.

The second graph shows the enlarged passband with 0.01dB/Div. Ripple is less than 0.01dB across the entire 110kHz passband. The third graph shows the group delay response demonstrating excellent phase linearity as a flat line of 169uS.



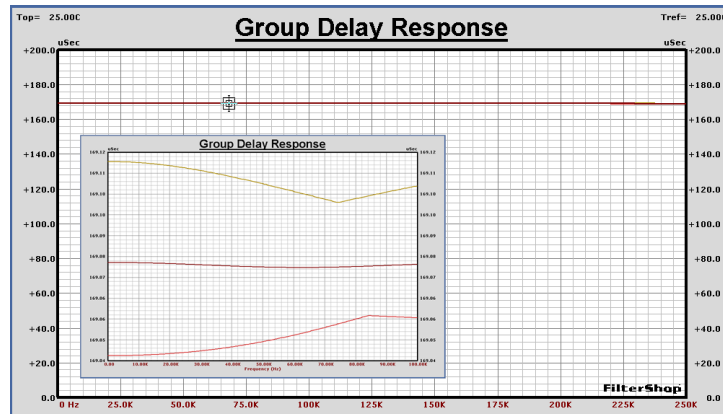
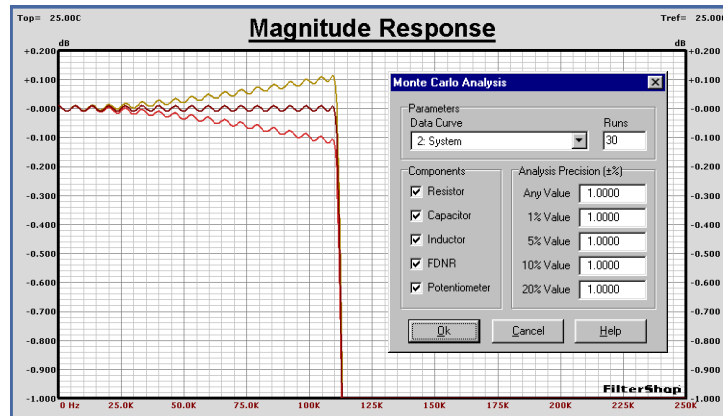
Complete Interpolation Filter Response



**■ Production Variance Analysis and Solutions**

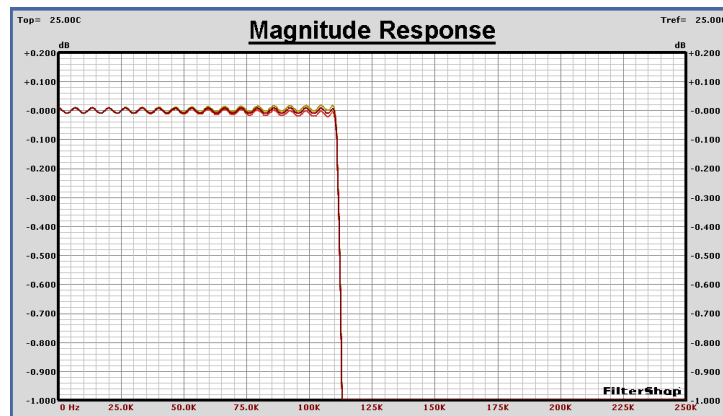
While the previous results easily met all of the design goals, the issue of production tolerance has not yet been addressed. The stability of a digital transfer function is extremely high, since they are finite calculations based on coefficients which would be identical from unit to unit. However, the analog circuitry would be constructed using components with a limited precision.

The *Processing / Monte Carlo Analysis* can be run on the complete circuit to examine some bounds on the possible variations. We will assume that 1% precision components are used for all of the resistor and capacitor components. The results below show the max/min limit curves based on 30 randomization runs.



The magnitude response shows up to a  $\pm 0.1\text{dB}$  deviation as the frequency approaches the high passband edge at  $110\text{kHz}$ . The group delay shows little relative effect, while the enlarged insert does show that small variations do indeed occur.

If we run the MC analysis again, this time with R/C precision of  $0.1\%$ , the graph below shows the magnitude response. In this case the variations fall within a  $\pm 0.02\text{dB}$  range. Still about twice what is desired.

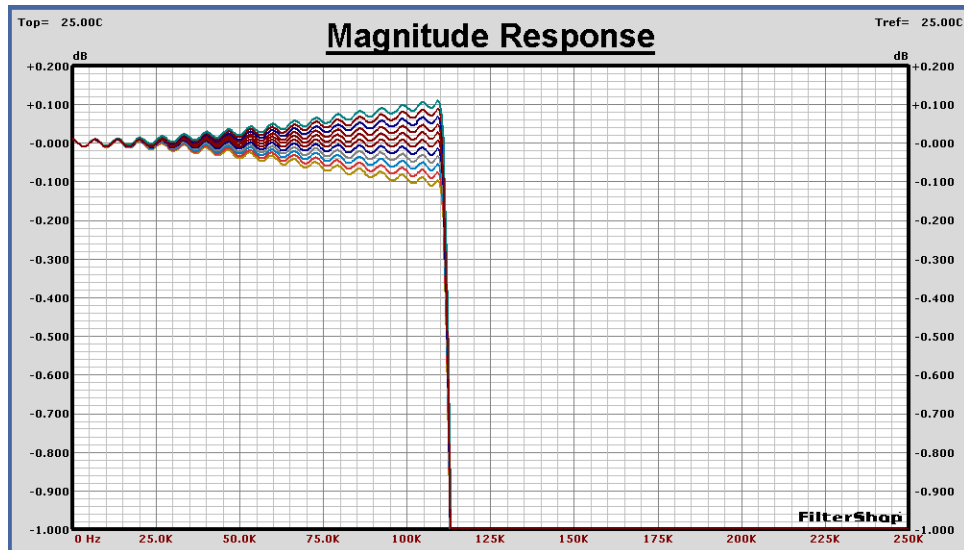
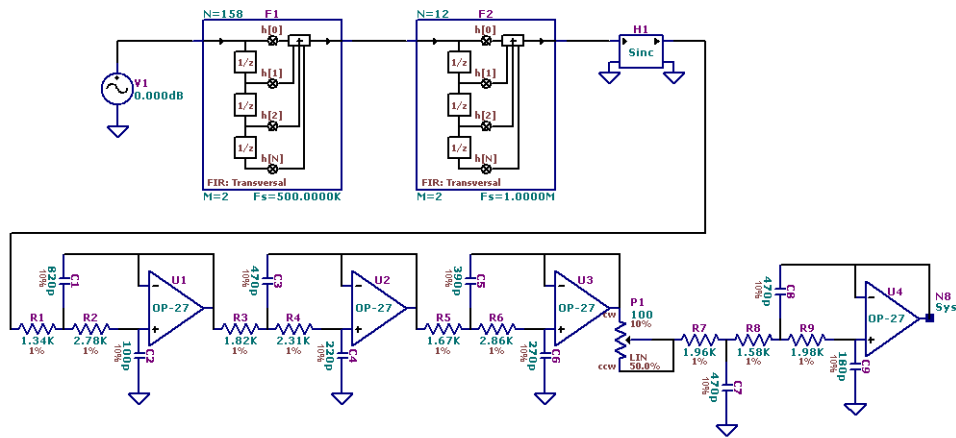


An alternative approach which does not require ultra high precision components is a *trim pot* for calibration. The sensitivity analysis showed that all of the R/C components had roughly the same unity sensitivity on controlling the response. For small changes in any component value, an almost linear relationship exists for changes at the output.

Although any of the 18 different R/C components could have value variations, they all amount to the same effect at the output for small changes and can be compensated with a single trim pot.

One of the resistors must be decreased in value slightly, and a pot placed in series. Almost any of the resistors could be used, and here R7 will be chosen. The original value of  $2\text{K}$  will be changed to  $1.96\text{K}$  and a  $100\ \Omega$  trim pot added in series.

The newly modified circuit schematic is shown below. The *Processing / Potentiometer Analysis* was run using 11 steps for autorotation. The magnitude graph below shows the family of 11 potentiometer curves. By adjusting this single pot, the flatness of the filter can be calibrated for any 1% component variations. This could easily be done in an automated fashion by two low/high test frequencies, and a programmable resistance.





**■ Summary**

The multirate FIR design dialog greatly simplifies the design of interpolators and other multirate filters. The optimum order for each stage is provided along with the determination of the edge frequencies. Many different configurations can quickly be obtained.

If minimum signal delay had been a requirement rather than linear phase, the FIR interpolation filters could have been easily designed for minimum phase by simply selecting that filter type in the FIR design dialogs.

The correction of both digital and analog transfer functions can be performed with high accuracy and relative ease. The finite bandwidth effects of the opamps themselves are included within the analog Lowpass. The entire mixed domain filter can be simulated and optimized together as a complete system.

This completes the Digital FIR Interpolator & Analog Lowpass design.

