

Application Note

10

Digital IIR Interpolator & Analog Lowpass

Highlights

Multirate IIR Design
Group Delay Compensation
Target Optimization
 $\text{Sin}(x)/x$ Correction
Cascade Analog Lowpass
Circuit Optimization

■ Design Objective

1:16 IIR Interpolation Filter
Fs: Input = 10kHz, Output=160kHz
4.25kHz Bandwidth, Linear Phase
0.025dB Passband Flatness
100dB Stopband Attenuation

This design will provide an example of how to integrate both the analog and digital simulation features of the program. A high performance interpolation filter will be produced. These filters are often used for DAC output reconstruction, and also in other situations where narrow-band low frequency filtering is required.

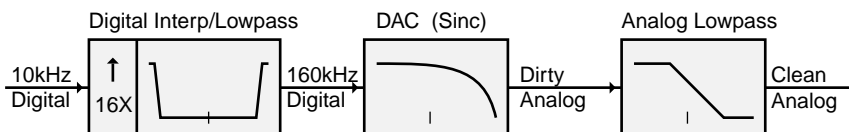
This example could arise from an instrumentation system, or general purpose data acquisition system, where the main internal processing data rate was 10kHz. The time domain data must now be converted back into analog via DAC.

If the 10kHz data was fed directly into the DAC, a highly selective analog elliptic filter would be required for reconstruction to remove the image band above the Nyquist frequency of 5kHz. Using multirate techniques, an oversampling reconstruction filter can be produced with greatly improved performance.

Interpolation is the process of upshifting the data (sampling) rate to a higher frequency. This then enables digital filtering based on the higher frequency. By increasing the data rate digital filtering can be applied directly at the main passband edge. The analog filter is then used to remove the much higher image products, allowing use of a wider transition band. In contrast to the previous approach, this method requires DSP and the use of a DAC with 16X higher speed capability.

■ Basic Filter System Description

A 1:16 total interpolation ratio will be used. Therefore the output DAC would need to support 160kHz data rates. The diagram below illustrates the mixed signal flow of the filter:



The Up-Convertor increases the sampling rate by adding zero samples between the input samples. This is a simple DSP operation, and a 1:16 convertor would add 15 zero samples between every original sample.

The digital Lowpass filter provides the critical cutoff near 5kHz. The image band then lies above 155kHz (160-5) as shown above. The DAC contains a sample & hold which produces a $\sin(x)/x$ (sinc) transfer function at 160kHz. After the DAC converts the signal to analog, the analog Lowpass filter then removes the high frequency image band.

The transition bandwidth for the analog Lowpass filter is now between 5kHz to 155kHz. This provides a much wider region to reach the high stopband attenuation.

The passband cutoff frequency for many digital systems is often 0.45 times the sampling frequency. This yields a 5% transition band. However for this IIR example the cutoff is 0.425 times the sampling frequency, providing a transition band of 7.5%. Allowing for a wider transition band enables the use of a smoother Lowpass corner with a much smaller group delay spike. This is helpful for the delay compensation.

The digital IIR filter is produce by transforming a highly selective analog prototype. This filter will not have linear phase by itself. To produce linear phase response, an additional Allpass compensation filter must be designed to complement the group delay of the IIR Lowpass filter.

For this design a linear frequency range from 0Hz - 10kHz will be used, or 0Hz - 160kHz so that all of the upper image bands will be visible. The Data Curves will be generated with 600 point resolution.

■ Multirate IIR Filters

The program does not provide a fully automatic method for the design of IIR filters, as is the case for FIR. While FIR multirate filters often have two or more stages, IIR realizations rarely use more than two.

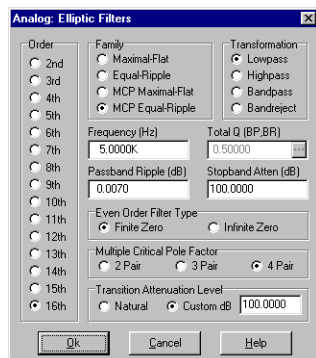
FIR filters benefit much more from multiple stages and by keeping the sampling rate close to the edge frequencies for each stage. For IIR filters this has much less importance, and in many cases using more stages simply reduces the efficiency. Unless the sampling ratio is very large, most designs use a single IIR stage.

For this 1:16 interpolator a single IIR stage will be used. The passband edge is 4.25kHz and the stopband edge is 5kHz. The first step is to select a suitable analog prototype that meets this criteria.

By analog standards, this is a very narrow filter and elliptic types are required. There are two basic family characteristics to choose from: maximally flat or equal ripple.

If a maximally flat family is used, it will be difficult to achieve a transition region small enough between the 4.25kHz and 5kHz edge frequencies. Therefore equal ripple families are indicated.

The standard equal ripple family has no problem providing very narrow transitions, but the group delay spike produced is very large. As somewhat of a compromise between the maximally flat and standard equal ripple, the MCP Equal Ripple family will be selected.



After a little trial & error with various elliptic designs, the final parameters used for the filter are: stopband attenuation 100dB, passband ripple of 0.007dB, and order of 16. An MCP factor of 4 identical poles was also used.

The custom transition level was set to 100dB and the corner frequency set to 5kHz. This is a neat trick that positions the filter edge right at our stopband.

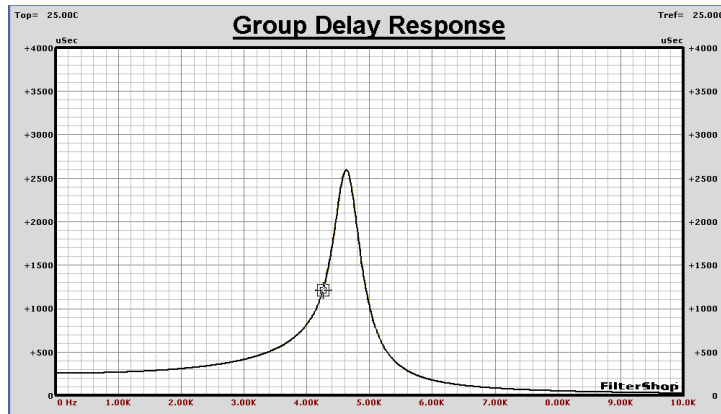
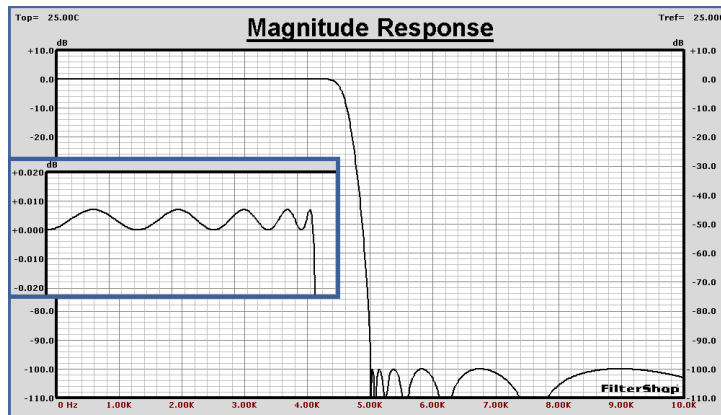
These parameters meet the transition criteria. The response is shown on the next page.

While the response rolls-off at a very high rate, the knee near 4.25kHz is nicely rounded. This helps to reduce the group delay spike. The response falls 100dB from 4.25kHz to 5kHz, while maintaining a passband ripple of 0.007dB.

Target Parameters												
TFB	Mag	Pha	Eqn Grp	FilterType	Ao (dB)	Fp, Fc (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Inv	Rev	M1	BR1	0.0000	4.6331K	8.4263	5.0082K	1.0000			
2	Inv	Rev	M1	BR1	0.0000	4.6331K	8.4263	5.0780K	1.0000			
3	Inv	Rev	M1	BR1	0.0000	4.6331K	8.4263	5.2390K	1.0000			
4	Inv	Rev	M1	BR1	0.0000	4.6331K	8.4263	5.5566K	1.0000			
5	Inv	Rev	M1	BR1	0.0000	4.1300K	3.1851	6.1941K	1.0000			
6	Inv	Rev	M1	BR1	0.0000	3.5903K	1.6815	7.5959K	1.0000			
7	Inv	Rev	M1	BR1	0.0000	2.8294K	0.9131	11.3922K	1.0000			
8	Inv	Rev	M1	BR1	0.0000	2.0905K	0.5484	32.2063K	1.0000			

The target parameters for the 16th order analog filter are shown here.

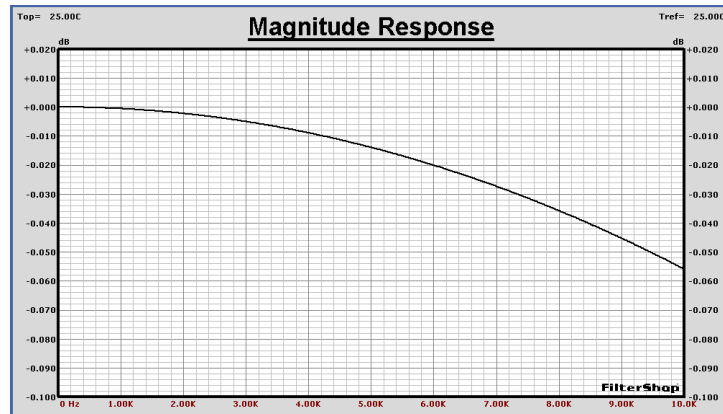
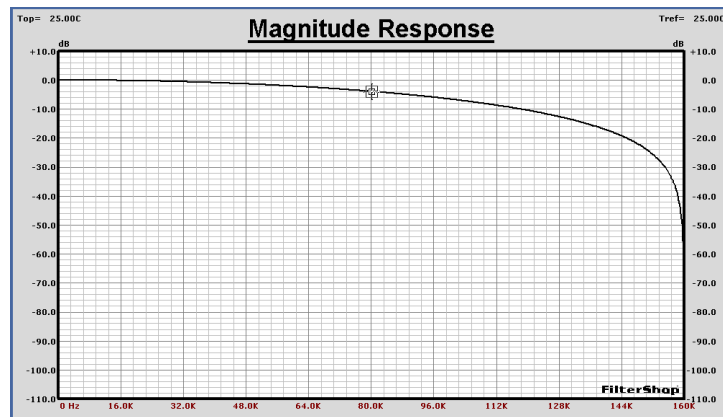
The magnitude response for the stop-band and passband regions are shown below, along with the group delay.



■ **DAC Sin(x)/x Transfer Function**

As mentioned previously we also wish to correct for a $\sin(x)/x$ (sinc) response based on a 160kHz frequency in the DAC. This can be trivially created in the Target Parameters using a single SINC filter type TFB with $F_s = 160\text{kHz}$. With the previous Lowpass TFBs shut off, the Sinc is shown below in the Black curve.

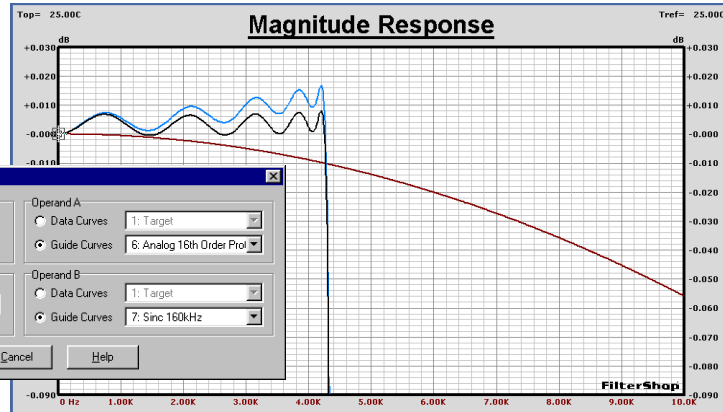
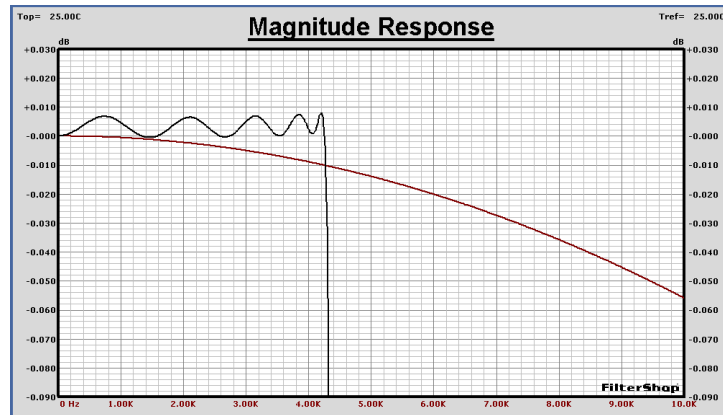
While the sinc response appears nearly flat in the 5kHz passband when viewed on the 160kHz scale, the response actually has a droop of almost -0.014dB at 5kHz as shown in the expanded 10kHz graph. If we are to meet the 0.025dB passband flatness specification, this must be included in the design. The Sinc response is copied to a Guide Curve for use as an optimizer objective.



■ Optimizing the Lowpass for Sinc Correction

We now wish to build the correction for the Sinc response into the previous 16th order analog Lowpass prototype. The first graph below shows the Sinc response in Red, and the Lowpass response in Black.

To optimize the response of the 16th order Lowpass function, an objective must be created which will not cause the previous Lowpass design to be destroyed. The Lowpass response will be divided by the Sinc response. This preserves the passband ripple character in the objective, but modifies it by the inverse of the Sinc transfer function. The Blue curve in the lower graph displays the new optimizer objective.



Binary Math Operations

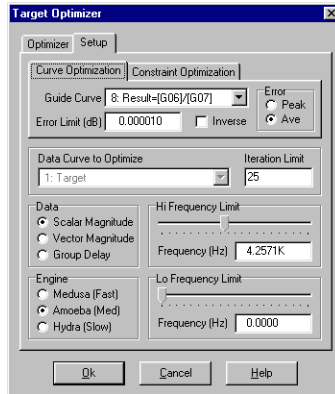
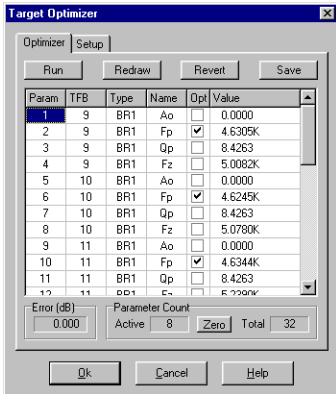
Operation:
 Mul : Result=A*B Add : Result=A+B
 Div : Result=A/B Sub : Result=A-B

Result:
 Guide Curve: 8: Result=(G06)/(G07)

Operand A:
 Data Curves: 1: Target
 Guide Curves: 6: Analog 16th Order Prot

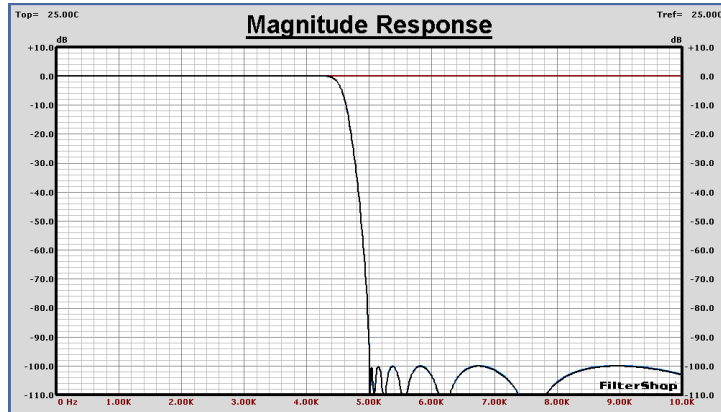
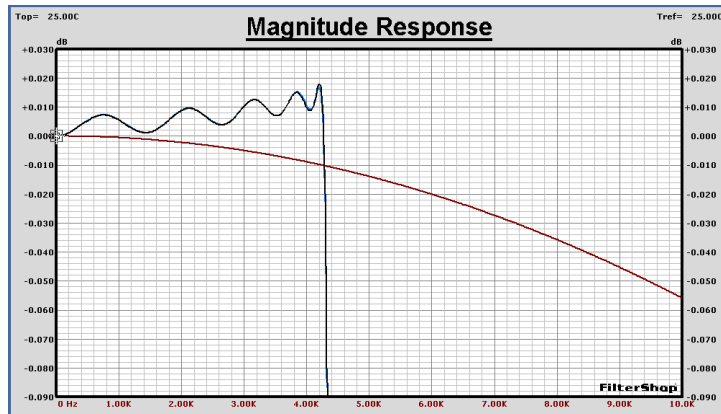
Operand B:
 Data Curves: 1: Target
 Guide Curves: 7: Sinc 160kHz

Buttons: [OK] [Cancel] [Help]



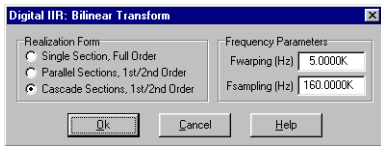
The *Target Optimizer* is now setup with the parameters shown here on the left. The frequency range will be limited to 0Hz - 4.25kHz, which is just the passband.

We can probably assume that this small change can be made using the eight Fp values of the BR1 sections. After optimization, the response below is produced. The transition/stopband are unchanged.

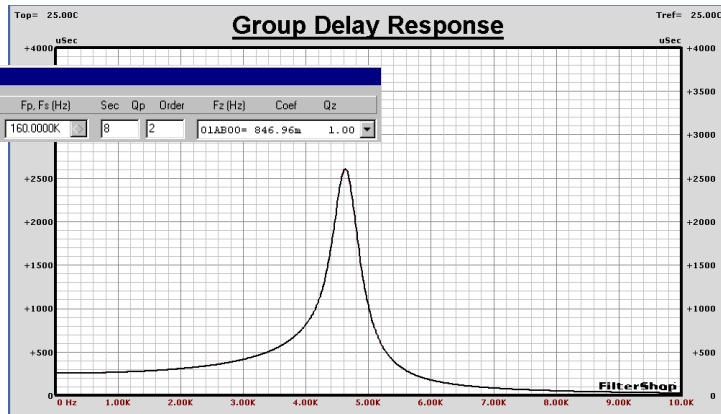
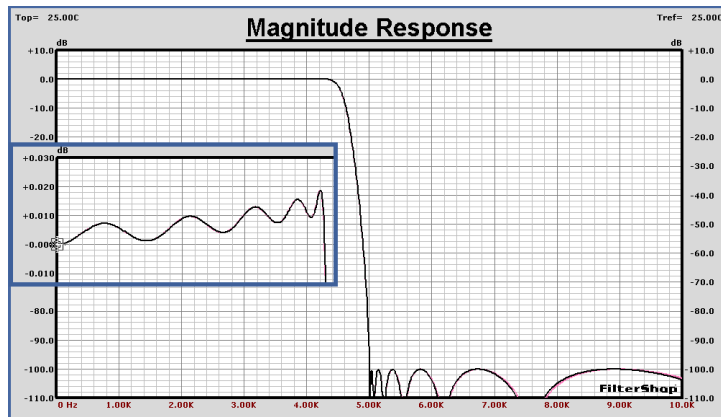


■ IIR Lowpass Transform

We will now transform the 16th order analog Lowpass prototype into its equivalent IIR form. The Bilinear transform works well on elliptic filters and will be used. The sampling rate is 160kHz and the warping frequency will be set to 5kHz. We do not want the stopband edge to move, so the 5kHz frequency is best. A cascade form will be selected. The dialog setup is shown here.



The response below shows the original analog prototype in Red, and the new IIR response in Black. The IIR transformation is virtually identical to the original analog. The IIR filter is now a single TFB in the Target Parameters as shown below.



TFB	Mag	Phs	Eqn	Gip	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
Invr	Rev	M1	IIR			0.0000	160.0000K	8	β	2	0.118000=	846.96m	1.00

■ Allpass Delay Compensation

An Allpass filter will now be designed to flatten the group delay of the Lowpass filter. The first obvious question is: What order of Allpass is required?

There is no uniform answer to this question for the general case. It depends on the shape of the group delay response to be corrected, and how much ripple we can tolerate in the final group delay.

A good rule of thumb is to expect the Allpass order to be similar to that of the response being corrected. The shape of our Lowpass group delay shows a peak which is about 10 times the 0Hz flat level (2500uS vs. 250uS). This is a fairly dramatic change in delay, and will require a larger Allpass order.

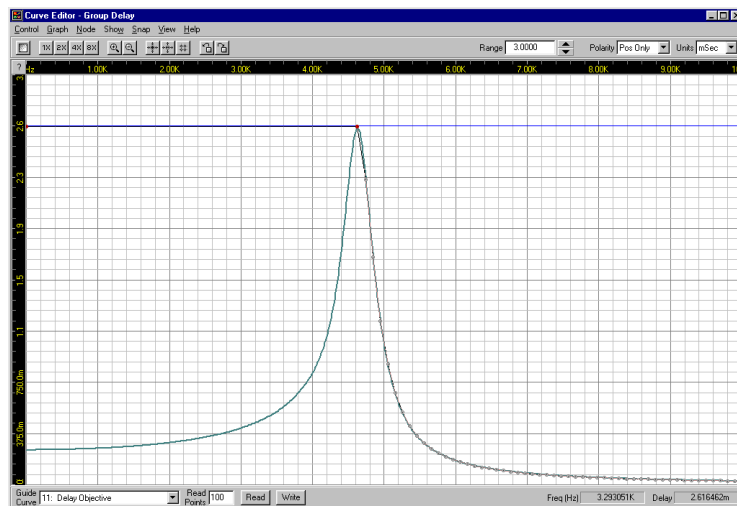
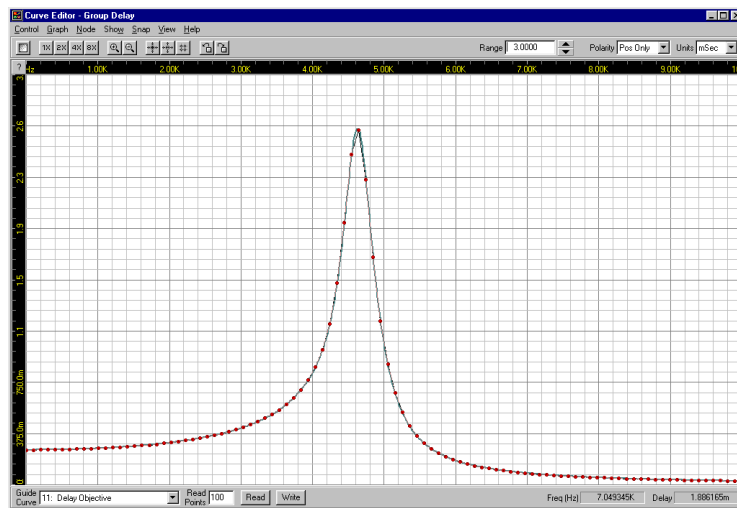
A 24th order Allpass filter will be attempted. This order is 150% larger than the Lowpass and should enable a low ripple delay to be produced.

The *Target Parameters* are now modified to include 12-AP2 sections as shown below. As initial values, all of the AP2 sections will have Fp=3kHz and Qp=0.7. The first TFB is the previously designed IIR containing the Lowpass filter.

Target Parameters													
TFB	Mag	Phs	Eqn Grp	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz	
1	Inv	Rev	M1	IIR	0.0000	160.0000K	8	2		0.1AB00= 846.96a	1.00		
2	Inv	Rev	M1	Sinc	0.0000	160.0000K		0.7071		1.3317K		1.0000	
3	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
4	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
5	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
6	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
7	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
8	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
9	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
10	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
11	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
12	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
13	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	
14	Inv	Rev	M1	AP2	0.0000	3.0000K		0.7000		704.8180K		1.0000	

The next step is to create a group delay objective curve for optimization. This is done by copying the Lowpass response to a Guide Curve, and then editing the Guide Curve in the *Curve Editor* utility.

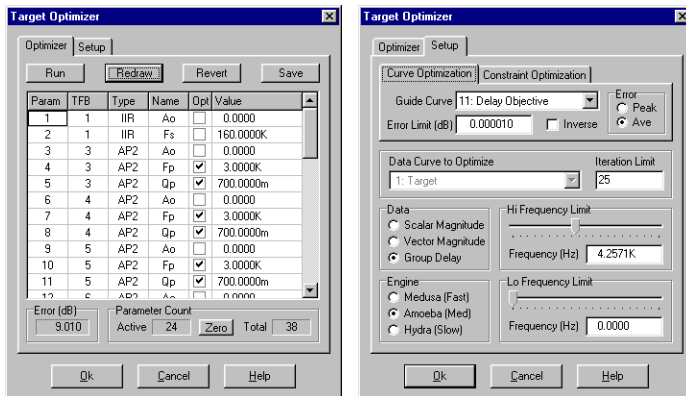
The first graph below shows the Curve Editor with the Lowpass group delay loaded. The curve was initially read in using 100 points. We wish to edit the lower portion of the curve below the peak for a flat line. As a simple guess, we will set the flat line equal to the top of the peak, about 2600uS. The lower points are deleted, and then edited as shown in the lower graph.



The combined group delay response of the Lowpass and Allpass is now shown below in Black, along with the objective curve in Green. Since we are only interested in flattening the delay in the passband, the optimization frequency range is limited to 0Hz to 4.25kHz.

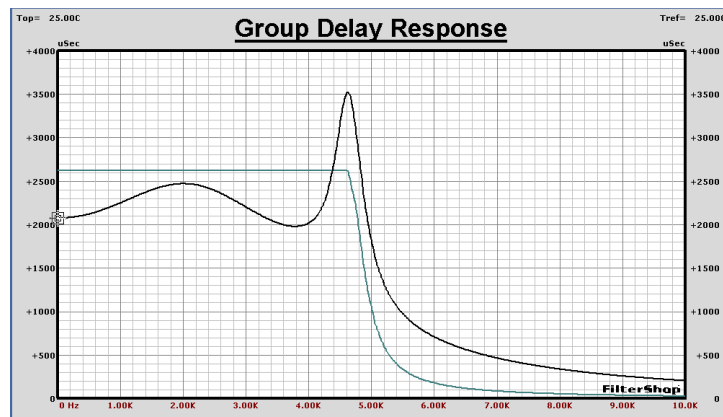
There will be 24 parameters optimized. These are the 12 pair of Fp and Qp values for the AP2 TFBs. The starting values of the AP2 sections are very crude. The first optimization will utilize *Average* error to get the parameters more in the ball park, and then reoptimized again using *Peak* error to trim the equal ripple response.

The dialogs below show the *Target Optimizer* setup.



The first graph on the following page displays the results after the first optimization using *Ave* error. The second graph shows the final results using *Peak* error.

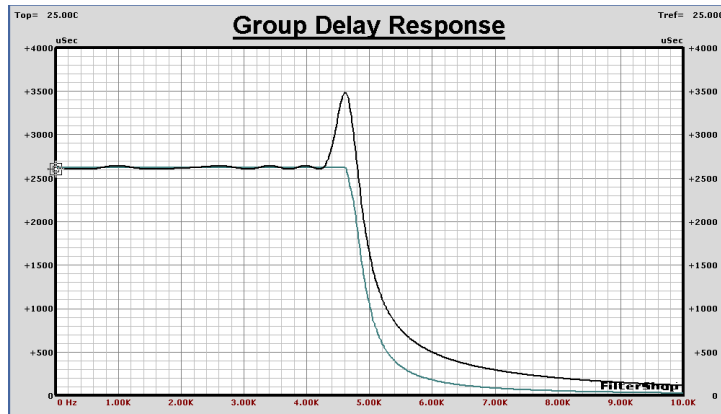
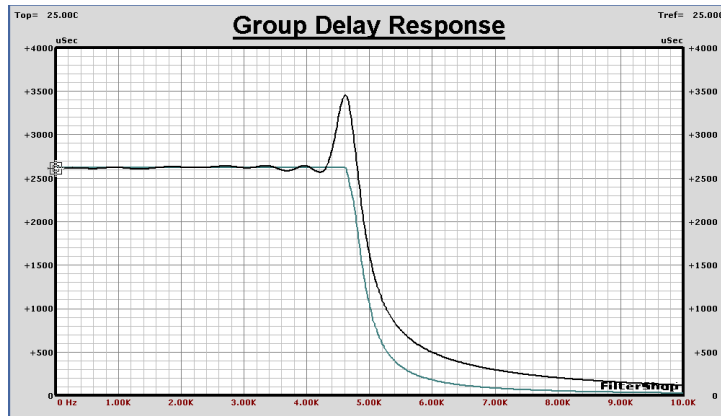
The final values for the 24th order Allpass filter are shown in the *Target Parameters* on the following page.



Target Parameters													
TFB	Mag	Phs	Eqn Gp	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz	
1	Inv	Rev	M1	IIR	0.0000	160.0000K	β	2		0.1AB00= 846.96m	1.00		
2	Inv	Rev	M1	Sinc	0.0000	160.0000K		0.7071		1.3317K		1.0000	
3	Inv	Rev	M1	AP2	0.0000	2.3349K		1.4788		704.8180K		1.0000	
4	Inv	Rev	M1	AP2	0.0000	2.8599K		0.7822		704.8180K		1.0000	
5	Inv	Rev	M1	AP2	0.0000	3.1332K		1.4895		704.8180K		1.0000	
6	Inv	Rev	M1	AP2	0.0000	2.7403K		0.6977		704.8180K		1.0000	
7	Inv	Rev	M1	AP2	0.0000	4.0350K		4.6622		704.8180K		1.0000	
8	Inv	Rev	M1	AP2	0.0000	2.8427K		2.0709		704.8180K		1.0000	
9	Inv	Rev	M1	AP2	0.0000	3.4837K		3.4166		704.8180K		1.0000	
10	Inv	Rev	M1	AP2	0.0000	1.8069K		1.1809		704.8180K		1.0000	
11	Inv	Rev	M1	AP2	0.0000	1.4337K		0.6891		704.8180K		1.0000	
12	Inv	Rev	M1	AP2	0.0000	2.7902K		0.8031		704.8180K		1.0000	
13	Inv	Rev	M1	AP2	0.0000	693.3119		0.5429		704.8180K		1.0000	
14	Inv	Rev	M1	AP2	0.0000	1.1426K		0.8658		704.8180K		1.0000	

The final result in the lower graph shows a ripple of $\pm 0.7\%$ across the entire pass-band. The remaining delay peak occurs in the transition band, and is unimportant for this design.

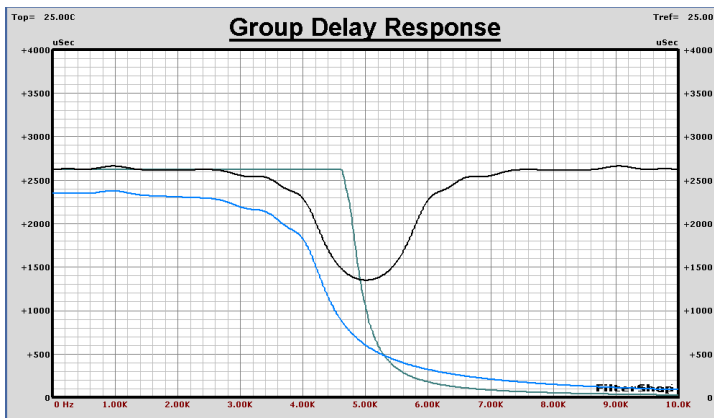
If desired, the ripple could be reduced or increased by using a higher or lower order for the Allpass filter.



■ Allpass IIR Transform

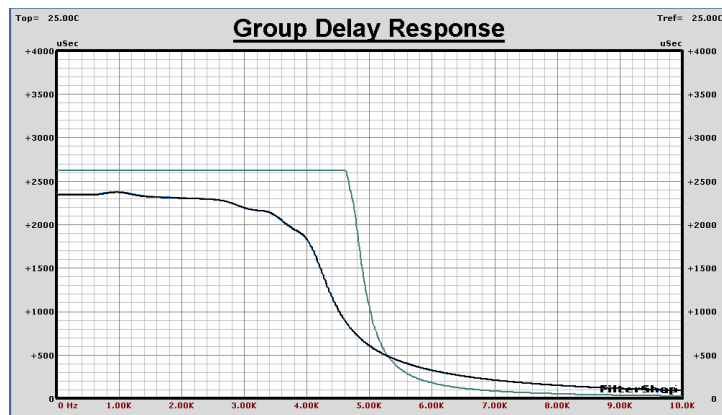
The analog Allpass filter must now be transformed into its IIR equivalent. There are two locations where this Allpass IIR could be placed: before or after the Lowpass/Interpolator IIR. If placed prior to the Lowpass IIR, the sampling rate is 10kHz. If placed after, the sampling rate is 160kHz.

It would be very desirable to use the Allpass IIR at the 10kHz sampling rate, since this would dramatically reduce the computations. Unfortunately this is not so easy. Because the Allpass response runs very near the 5kHz Nyquist frequency, substantial warping occurs. This severely damages the optimized Allpass response.



Both the Bilinear and Matched-Z transforms were attempted. The Bilinear solution blows-up completely, and the Matched-Z is shown here.

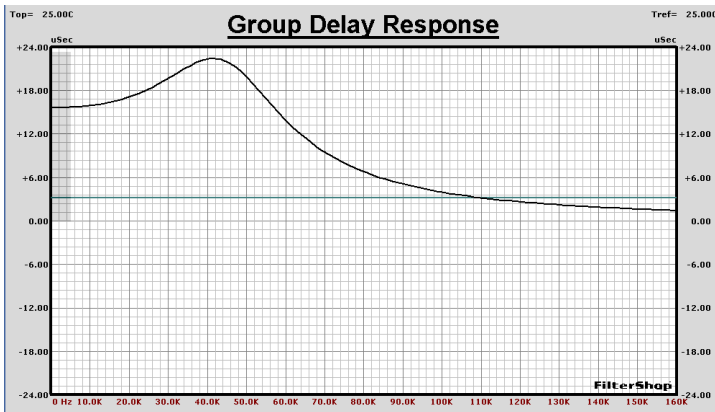
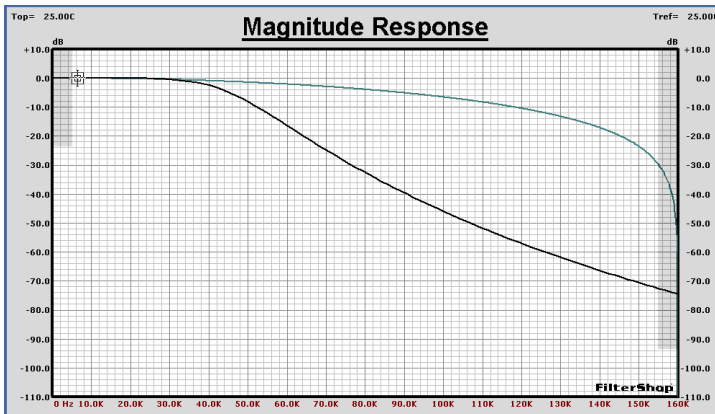
The graph below shows the Allpass prototype transformed using the 160kHz sample rate. The Black curve is virtually on top of the Blue prototype. For this example the 160kHz IIR version will be used.



■ Analog Reconstruction Filter Target

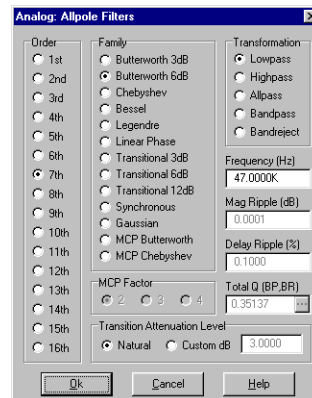
A filter must be placed after the DAC to remove the image near the 160kHz sample rate. This of course must be an analog filter. The $\sin(x)/x$ loss of the DAC sampling provides about 30dB of attenuation at the lower 5kHz image edge at 155kHz as shown below. This leaves about 70dB of attenuation that must be provided by the analog filter.

One might first think that a flat group delay family would be required. However with an interpolation ratio of 1:16 the transition bandwidth is wide enough that other filters can also be used. If the corner frequency of the filter is high enough, the group delay at low frequencies will be nearly flat for many filter families.



A Butterworth-6dB was chosen since this provides a compromise between the Bessel and standard Butterworth-3dB. A 7th order LP with $F_o=47\text{kHz}$ was chosen. The response is shown here in Black.

The group delay filter response in the gray region at 5kHz is virtually flat. The magnitude response in this same 5kHz range is less than 0.000005dB.



■ Analog Circuit Realization

To produce the 7th order analog Lowpass, a cascaded two stage design will be used consisting of 4th and 3rd order sections. The current Target Parameters for this filter are shown below.

Target Parameters													
TFB	Mag	Phs	Eqn	Gip	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Inv	Rev	M1		LP2	0.0000	47.0000K	1.3066			1.3317K	1.0000	
2	Inv	Rev	M1		LP2	0.0000	47.0000K	1.0000			1.6479M	1.0000	
3	Inv	Rev	M1		LP2	0.0000	47.0000K	0.5412			1.4317M	1.0000	
4	Inv	Rev	M1		LP1	0.0000	47.0000K	0.5412			1.5813M	1.0000	

There are three 2nd order TFBs and one 1st order TFB associated with the filter. We can choose to pair these up anyway we like to form the 4th and 3rd order targets. In order to produce a circuit with the best possible sensitivity, it is a good idea to pair low Q sections with higher Q sections.

The highest Q TFB is #1 so this will be paired with TFB #4 which is the 1st order TFB. This will form the 3rd order section. The 4th order section will be produced using TFB #2 and #3 with Q values of 1.0 and 0.5412. After a little rearranging the 4th and 3rd order sections are shown below.

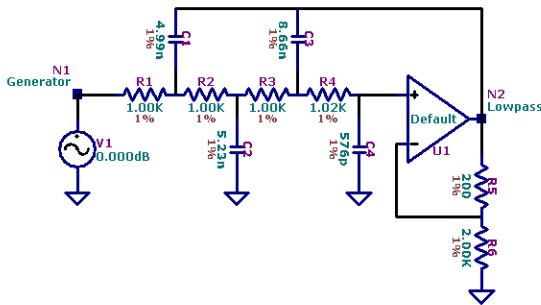
Target Parameters													
TFB	Mag	Phs	Eqn	Gip	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Inv	Rev	M1		LP2	0.0000	47.0000K	1.3066			1.3317K	1.0000	
2	Inv	Rev	M1		LP2	0.0000	47.0000K	1.0000			1.6479M	1.0000	
3	Inv	Rev	M1		LP2	0.0000	47.0000K	0.5412			1.4317M	1.0000	
4	Inv	Rev	M1		LP1	0.0000	47.0000K	0.5412			1.5813M	1.0000	
5	Inv	Rev	M1		LP2	0.0000	47.0000K	1.3066			1.3317K	1.0000	
6	Inv	Rev	M1		LP2	0.0000	47.0000K	1.0000			1.6479M	1.0000	
7	Inv	Rev	M1		LP1	0.0000	47.0000K	0.5412			1.5813M	1.0000	

Each of the two sections will be enabled/disabled alternatively to design each respective circuit section individually. After that they will be combined into the complete 7th order circuit.

■ Analog Filter Circuit: 4th Order Section

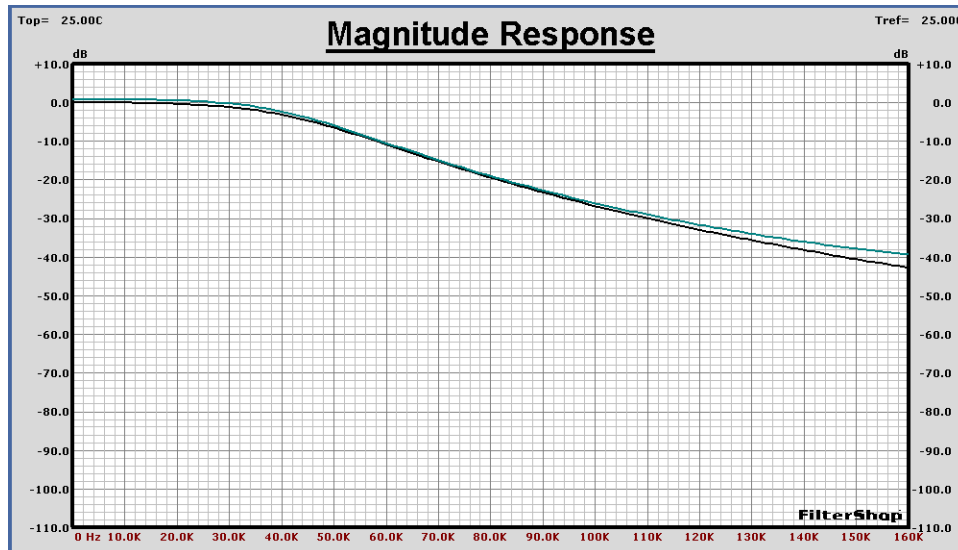
The LP4_SK1 synthesis circuit is loaded, and only TFBs #2 and #3 enabled. The 4th order Sallen-Key circuits can be tricky to design. Enough gain must be used to obtain a solution, but too much gain raises the component sensitivity. For this design the preset values of Gain = 1.1, 1K, and 2K were used for Synthesis. The circuit and its response is shown below as compared to the ideal target.

The circuit has a gain of 1.1 (0.83dB) where the target is at 0dB, and at high frequencies the curves begin to diverge. This is due to the limited bandwidth of the opamp. It will be important to simulate the circuit with a specific opamp.



To change the gain back to unity, R1 will be increased slightly and a resistor added to ground. We will also want to use 10% standard values for the capacitors. The opamp will be changed to a 5534 type.

The Target curve is copied to a Guide Curve, and then the revised circuit is optimized.

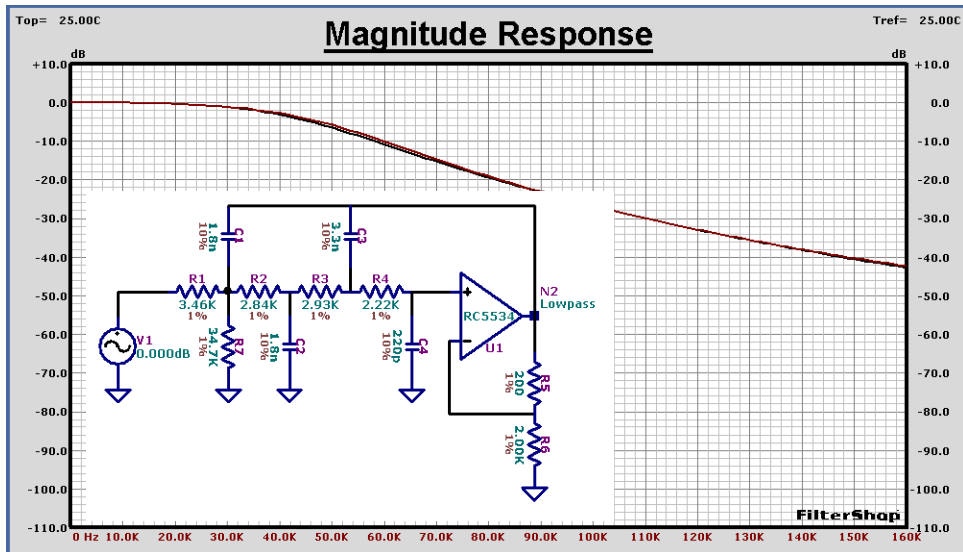
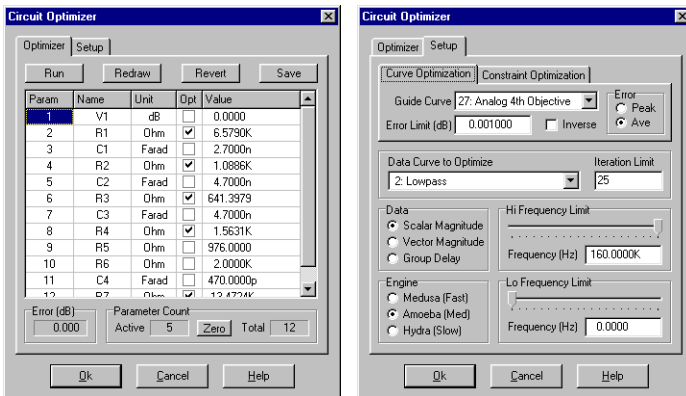


The optimization frequency range is initially the entire 160kHz bandwidth, with all components at 1% precision. Compensation for the opamp finite bandwidth will require changes in the capacitor values. In order to determine what the best range is for the capacitors, all of the components are optimized except for R5, R6.

The result of the optimization is a very close match to the curve everywhere. The capacitors are then changed to 10% precision, disabled for optimization, and then the circuit reoptimized again using the resistors.

When this is completed, the frequency range is decreased to a 20kHz bandwidth to maximize the fit in the critical passband region.

The 5kHz region now matches the target to within 0.004dB. The final results are shown below.

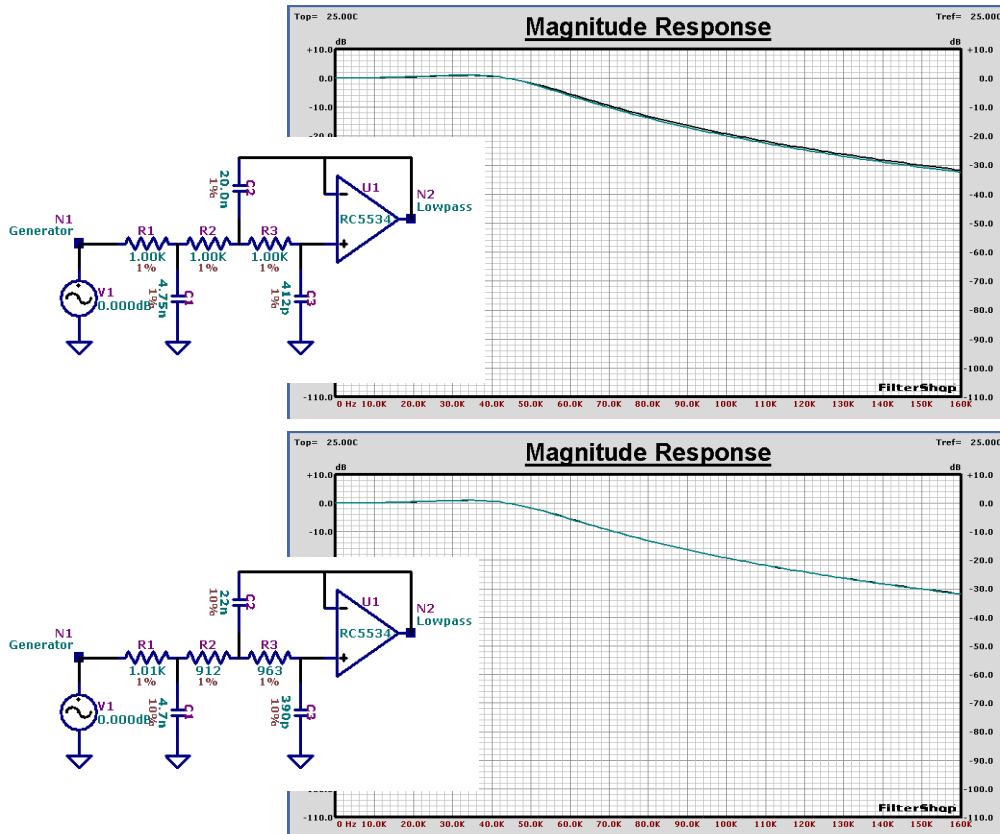


■ Analog Filter Circuit: 3rd Order Section

The LP3_SK1 synthesis circuit is loaded, and only TFBs #6 and #7 enabled. The synthesis for this circuit is straightforward. The synthesis preset resistor is set to 1k. The resulting circuit and response is shown below.

As with the previous circuit, the finite bandwidth of the opamp causes some difference in the response at high frequencies. This circuit will also be optimized.

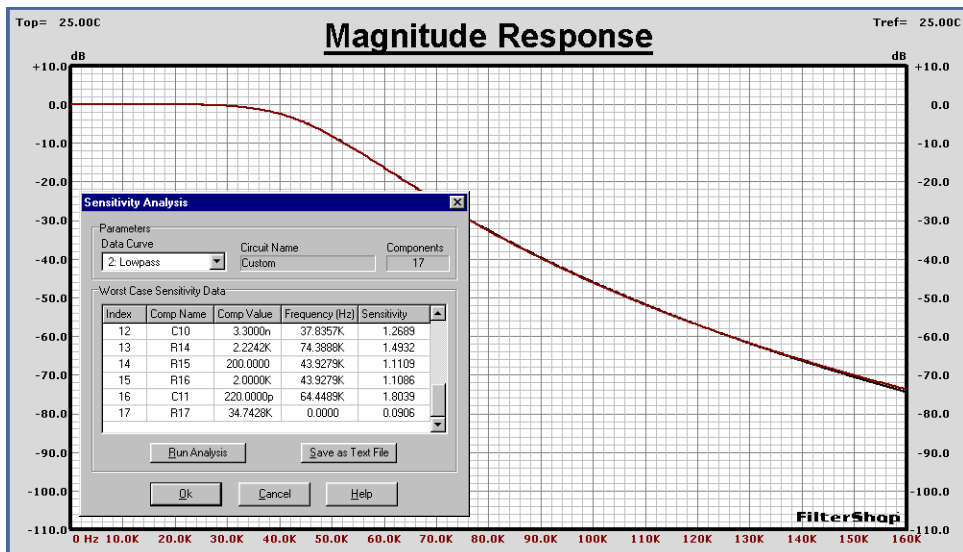
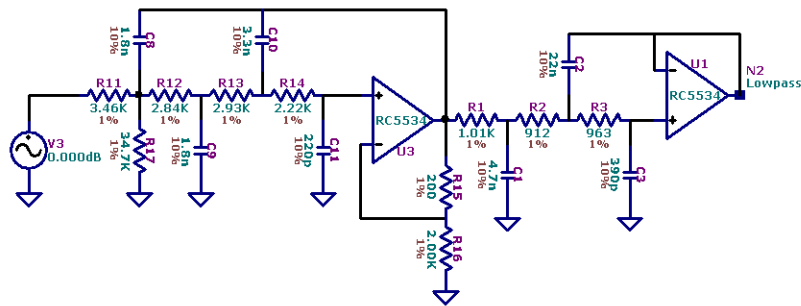
The capacitors are first changed to 10% standard values, and this time just the three resistors are optimized. After the circuit is optimized the response is virtually on top of the ideal 3rd order target everywhere.



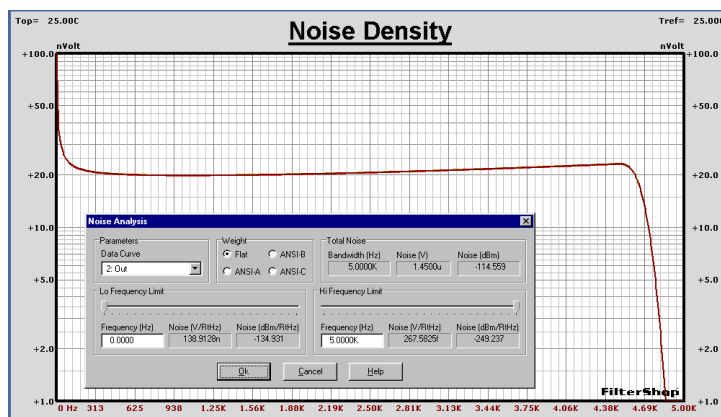
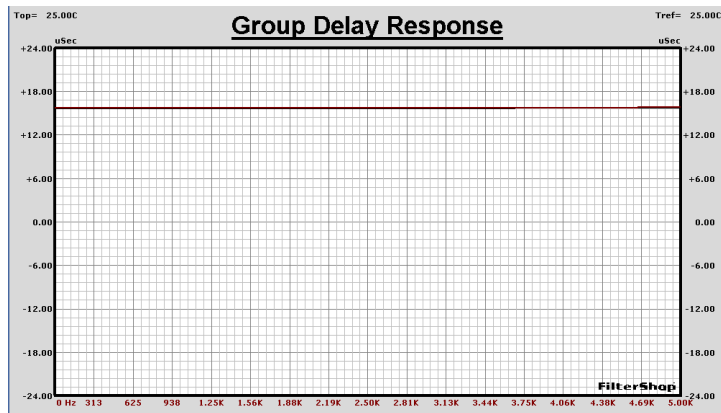
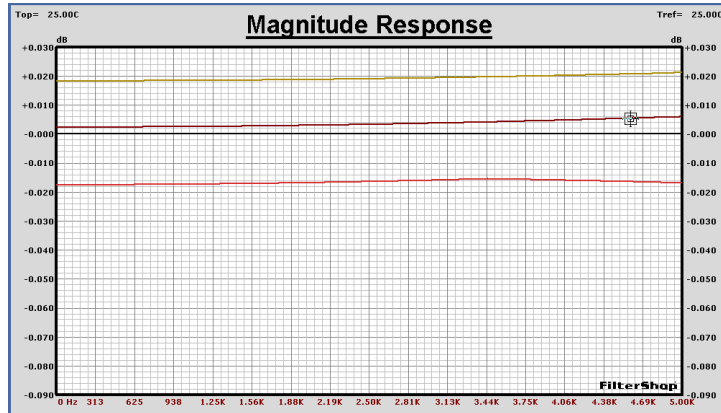
■ Complete Analog Filter Circuit

The 3rd and 4th order sections are now combined to form the complete analog filter. The circuit and its response is shown below. Sensitivity analysis shows excellent results with the worst case S value of 1.8 for C11. Most of the other values are below 1.5.

The detailed passband response is shown on the following page, along with a Monte Carlo analysis assuming 1% component tolerance. The Max/Min curves are between $\pm 0.02\text{dB}$ and the group delay is very flat. The broadband noise shows about 1.5uV (-114dBm) across the 5kHz bandwidth.



Complete Analog 7th Order Filter Passband Response

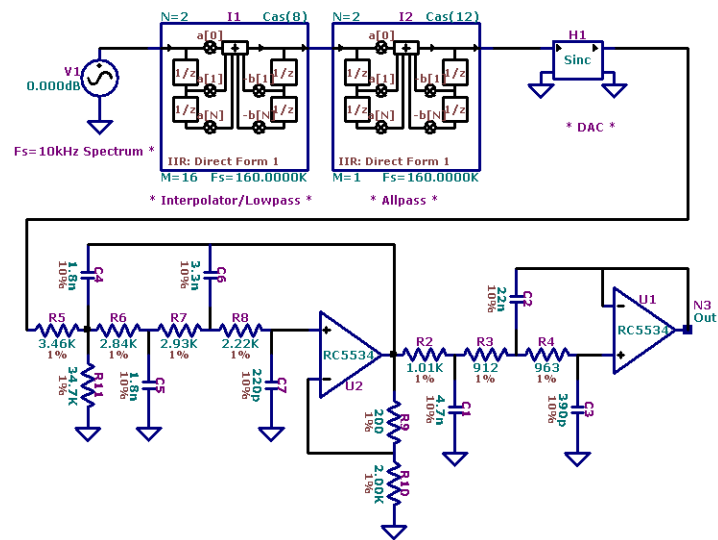


■ Complete Interpolator Filter Simulation

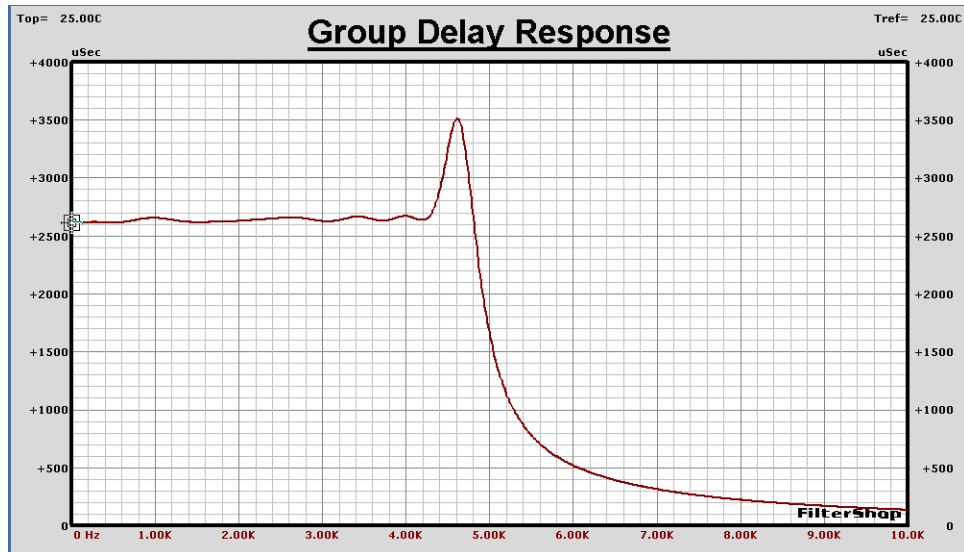
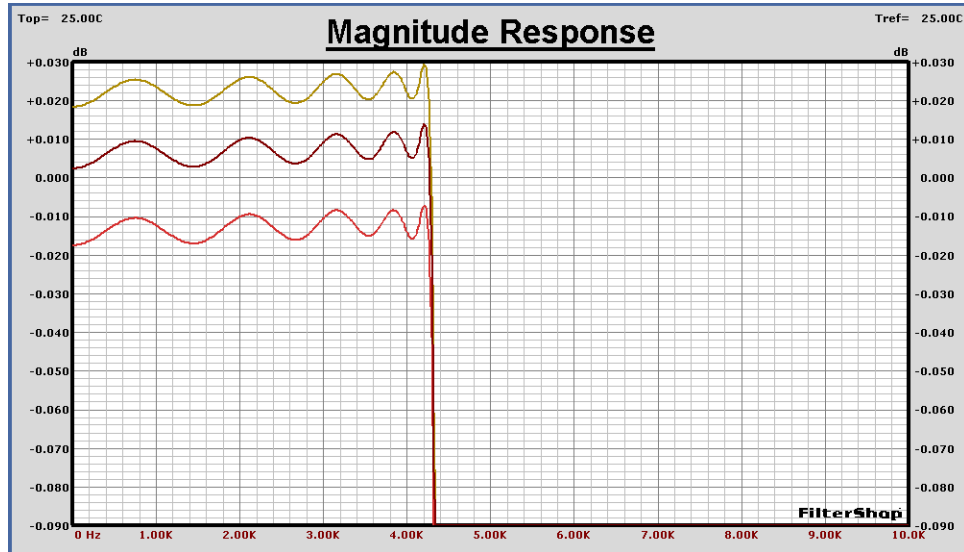
The entire analog/digital interpolation filter design can now be verified. A circuit is created as shown below to provide the frequency domain simulation. The two IIR filters follow the generator which contain the Lowpass and Allpass transfer functions. The target coefficient data is transferred into the components. An H transfer function component is used and setup as the Sinc function at 160kHz to represent the DAC, and the 7th order analog Lowpass filter follows.

The two graphs on the following page display the results. The first graph shows the passband frequency range from 0Hz to 10kHz with 0.01dB/Div. Monte Carlo Max/Min response is also shown.

The average base gain is +0.005dB with a range of about ±0.025dB including ripple. The group delay is flat across the passband with a value of 2.6mS.



Complete Interpolator Filter System Passband Response



■ Summary

The design of multirate IIR filters is very similar to that of ordinary IIR filters. In this example linear phase was the objective, and group delay correction was utilized. If only a minimum phase design was sought, the Allpass IIR could be readily eliminated.

The correction of both digital and analog transfer functions can be performed with high accuracy and relative ease. The mixed domain filters can also be simulated together as a complete system.

This completes the Digital IIR Interpolator & Analog Lowpass design.

