

Application Note

9

Digital FIR Decimator & Analog Lowpass

Highlights

Multirate FIR Design
Cascade Analog Lowpass
Circuit Optimization
Comb Filter Correction
Sin(x)/x Correction

■ Design Objective

16:1 FIR Decimation Filter
Fs: Input = 160kHz, Output=10kHz
4.5kHz Bandwidth, Linear Phase
0.025dB Passband Flatness
100dB Stopband Attenuation

This design will provide an example of how to integrate both the analog and digital simulation features of the program. A high performance decimation filter will be produced. These filters are often used for ADC input antialiasing, and also in other situations where narrow-band low frequency filtering is required.

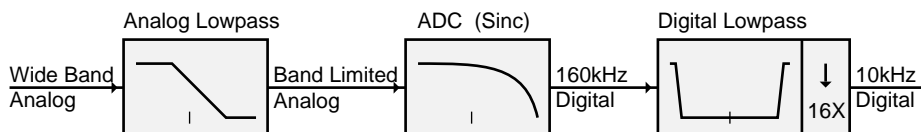
This example could arise from an instrumentation system, or general purpose data acquisition system, where the main internal processing data rate was 10kHz. The analog input data must be converted into digital via ADC and filtering.

If the 10kHz data was taken directly from the ADC, a highly selective analog elliptic filter would be required for antialiasing to remove any high frequency content above the Nyquist frequency of 5kHz. Using multirate techniques, an oversampling architecture can be produced with improved performance.

Decimation is the process of downshifting the data (sampling) rate to a lower frequency. This then enables digital filtering based on the higher frequency. By increasing the data rate digital filtering can be applied directly at the main passband edge. The analog filter is then used to remove the much higher image products, allowing use of a wider transition band. In contrast to the previous approach, this method requires DSP and the use of an ADC with 16X higher speed capability.

■ Basic Filter System Description

A 16:1 total decimation ratio will be used. Therefore the input ADC would need to support 160kHz data rates. The diagram below illustrates the mixed signal flow of the filter:

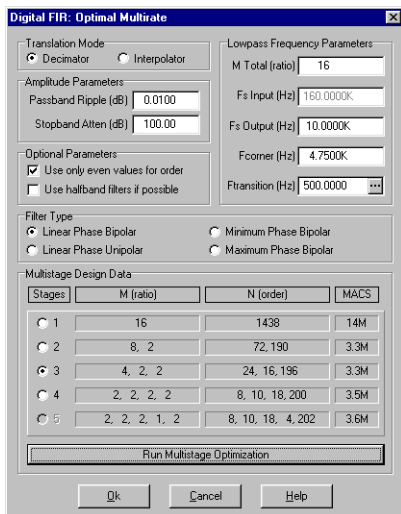


The down-converter at the end of the FIR filter decreases the sampling rate by simply dropping 15 out of every 16 FIR computations. Of course the 15 computations which would be dropped are actually never computed. Polyphase structures are very suited to this process and are commonly used.

The digital Lowpass filter provides the critical cutoff near 5kHz. The image band then lies above 155kHz (160k-5k) as shown above. Prior to the ADC converting the analog signal, the analog Lowpass filter removes the high frequency image band. The ADC contains a sample & hold which produces a $\sin(x)/x$ (sinc) transfer function at 160kHz.

For this design different linear frequency ranges from 0-5kHz to 0-160kHz will be used, depending on the sample rates associated with the different stages of the filter. Viewing filter response in the presence of decimation is generally more difficult than for interpolation. This is especially true for multistage decimators.

Most of the curves will be generated with 500 point resolution. However during optimization the data points are reduced to 100 for a reduction in processing time.



■ Multirate FIR Filters

We now wish to introduce one of the Target system FIR filter design dialogs: *Digital-FIR: Optimal Multirate*. This dialog greatly simplifies the design of FIR interpolators and decimators.

The parameters in the dialog are setup as shown here on the left. The Decimator selection is chosen, 0.01dB passband ripple, 100dB stopband attenuation, the M ratio set to 16, Even Order selected, and the output sampling rate set to 10kHz.

In choosing the corner and transition frequency, we click the small [...] button and enter the edge frequencies of 4.5kHz and 5kHz. If we wish to have clean transition bands, the full attenuation must be reached at 5kHz. The resulting corner and transition frequencies are 4.75kHz and 500Hz respectively.

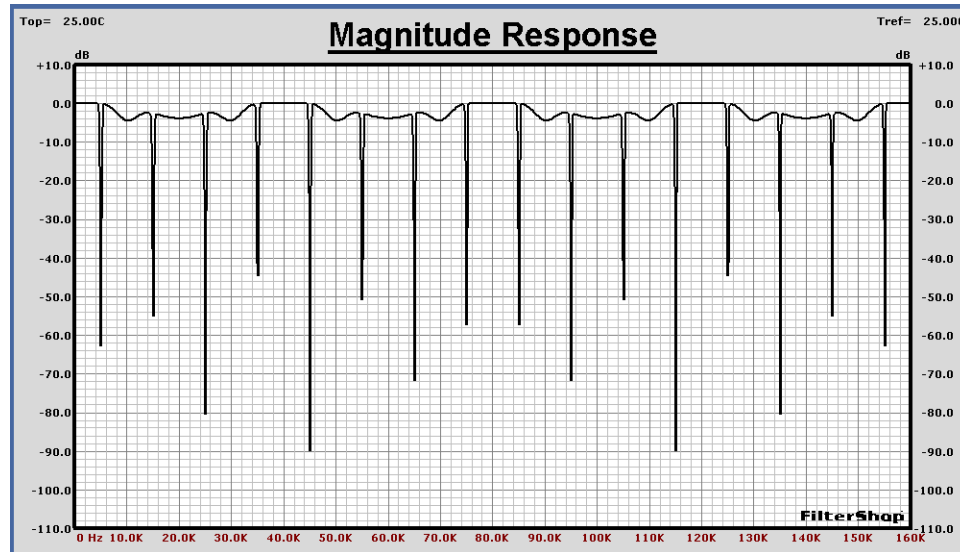
Note that the *Multistage Design Data* table is currently empty.

Click the *Run Multistage Optimization* button. The table is now filled with four rows of active data. Each row represents a different possible decimator configuration. Each contains a different number of decimator stages. The best order for each stage has been optimized by the dialog, with the computational requirements listed in the last column *MACS* (multiply accumulate per second).

Note that the first row shows a single stage decimator (16:1) and requires a very large FIR filter of order $N = 1438$. The *MACS* value is 14M. The 3rd row shows a three stage design with much smaller FIR filters and a *MACS* value of 3.3M. This is less than 1/4 the single stage design.

The multirate optimizer has already chosen this three stage design since it is the most efficient (lowest *MACS* value). Actually both the two stage and three stage show very similar *MACS* values. We will go with the default three stage design.

When the *Ok* button is clicked, all three FIR filters will be designed and loaded into the Target Parameters. The results are shown on the following page.

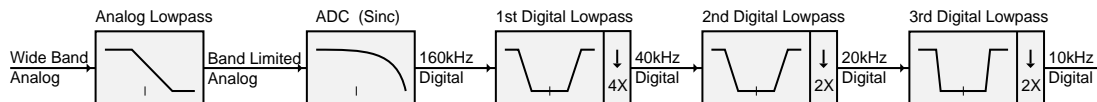


Target Parameters													
TFB	Mag	Phs	Eqn	Grp	FilterType	Ao (dB)	Fp, Fz (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Invl	Rev	M1		FIR	0.0000	160.0000K			24		H(0000) = 135.7668u	
2	Invl	Rev	M1		FIR	0.0000	40.0000K			16		H(0000) = -1.4229m	
3	Invl	Rev	M1		FIR	0.0000	20.0000K			196		H(0000) = -10.7514u	

Three FIR TFBs are shown in the Target Parameters dialog. Their combined response is shown above. Note that the sampling frequency F_s values of each stage are different. The orders are: 24, 16, and 196.

The magnitude response shown above contains the effects of three cascaded decimators, viewed in a linear system model where all transfer functions are multiplied together. Decimation is a resampling process, and tends to hide the filtering which took place prior to the decimation. As such we shall investigate the response of the filters both with and without decimation.

We will now examine the response of each individual stage separately.



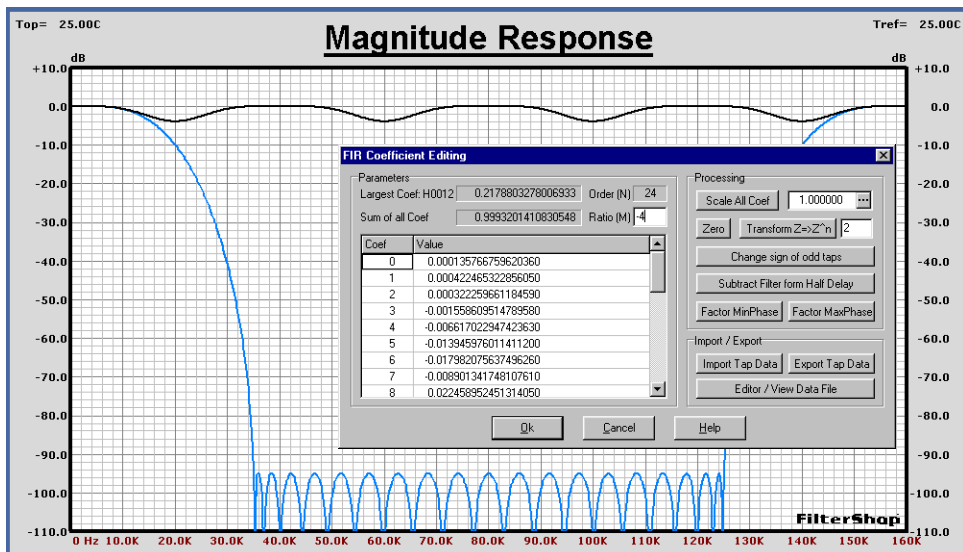
■ FIR / Decimator 1st Stage: 160kHz -> 40kHz (4:1)

The 2nd and 3rd TFBs are shut off, leaving just the first FIR filter/decimator. The effect of decimation can be controlled from the FIR Coefficient Editing dialog as shown below with the Ratio parameter (M). When the value is -4, a 4:1 decimator is produced. We can change the value to 1 to produce a normal FIR filter.

The Black curve below shows the output with decimation, while the Blue curve shows the FIR filter without decimation. The edge frequencies of the filter are 5kHz and 35kHz. The stopband attenuation came up a bit shy of the desired 100dB, at about 95dB. This is a common occurrence when designing optimal filters. The FIR order prediction for optimal filters is based on an *estimate*. The actual response is unknown until the final result is obtained by numerical solution.

The Black curve shows a periodicity of 40kHz, which is of course the output sampling rate of the decimator. Also included in the Black curve is aliasing from the transition bands of the actual FIR filter response. This is clearly evident in the level at 20kHz which is higher in the Black curve than the Blue curve.

Aliasing generally occurs in many decimator designs, but it is limited to the frequency regions outside the desired final bandwidth (in this case 5kHz).

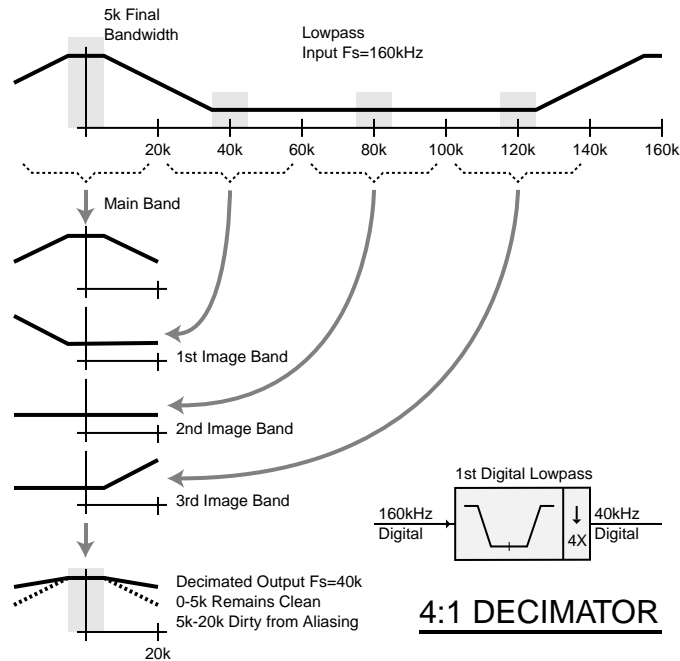


Decimation causes the higher frequencies to fold back into the output spectrum. This folding is based on the output sample rate, in this case 40kHz. In order to keep the desired bandwidth of 0Hz - 5kHz clean, the filter must remove all higher frequency regions which would fold back across the 5kHz band.

The diagram below shows how the aliasing occurs in this 4:1 decimator. The top graph shows the response of the 1st FIR filter. The $\pm 5\text{kHz}$ gray boxes centered at multiples of 40kHz indicate the regions which would fold back on the main output spectrum. These areas must be fully attenuated.

The 1st and 3rd image bands alias into the sides of the main output spectrum above 5kHz. The 2nd image band resides in the middle of high FIR filter attenuation and causes no aliasing.

The final output spectrum is thus dirty from 5kHz to 20kHz, but remains clean in the 0Hz to 5kHz range of interest.

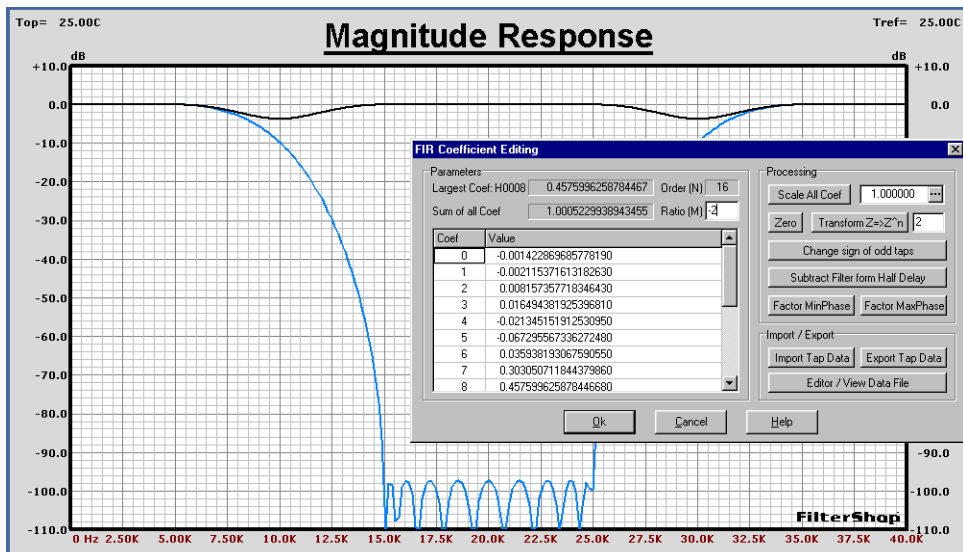


■ FIR / Decimator 2nd Stage: 40kHz -> 20kHz (2:1)

The 1st and 3rd TFBs are shut off, leaving just the second FIR filter/decimator. As before, the effect of decimation can be controlled from the FIR Coefficient Editing dialog as shown below with the Ratio parameter (M). When the value is -2, a 2:1 decimator is produced. We can change the value to 1 to produce a normal FIR filter.

The Black curve below shows the output with decimation, while the Blue curve shows the FIR filter without decimation. The edge frequencies of the filter are 5kHz and 15kHz. Since the highest sampling rate involved with this stage is 40kHz, the system frequency range was changed to match.

The Black curve shows a periodicity of 20kHz, which is of course the output sampling rate of the decimator. Also included in the Black curve is aliasing from the transition bands of the actual FIR filter response. This is clearly evident in the level at 10kHz which is higher in the Black curve than the Blue curve.

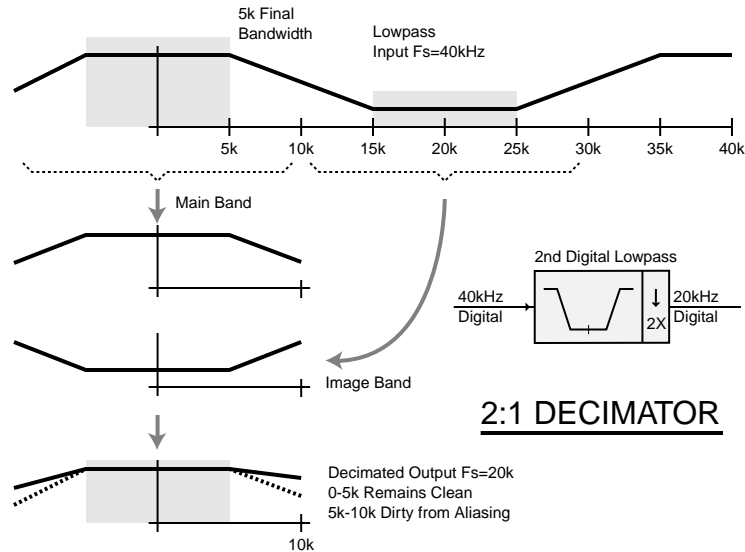


The output sampling frequency for this stage is 20kHz. In order to keep the desired bandwidth of 0Hz - 5kHz clean, the filter must remove all higher frequency regions which would fold back across the 5kHz band.

The diagram below shows how the aliasing occurs in this 2:1 decimator. The top graph shows the response of the 2nd FIR filter. The $\pm 5\text{kHz}$ gray boxes centered at multiples of 20kHz indicate the regions which would fold back on the main output spectrum. These areas must be fully attenuated.

For a 2:1 decimator there is only a single image band, and it causes aliasing in both sides of the output spectrum. But again this only occurs above 5kHz, outside the final band of interest.

The final output spectrum is thus dirty from 5kHz to 10kHz, but remains clean in the 0Hz to 5kHz range of interest.



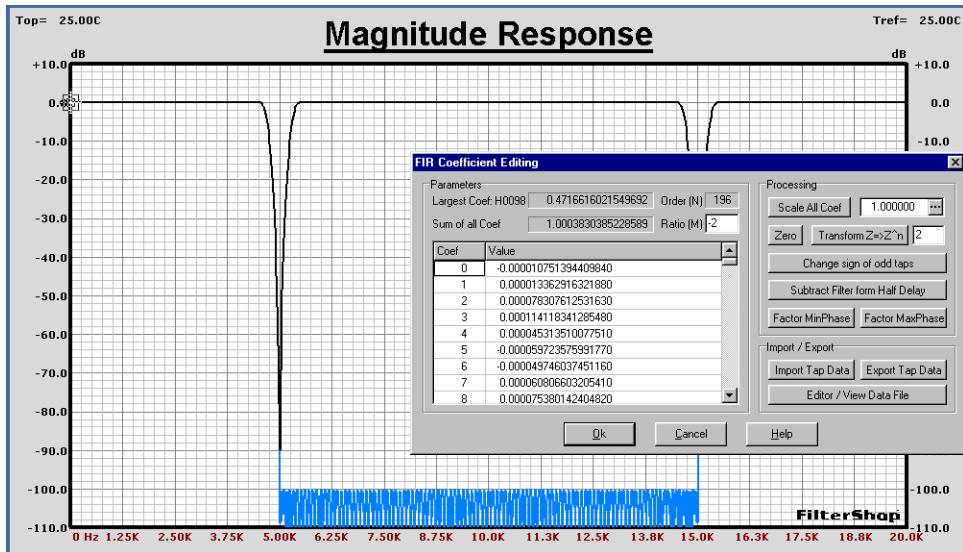
■ FIR / Decimator 3rd Stage: 20kHz -> 10kHz (2:1)

The 1st and 2nd TFBs are shut off, leaving just the third FIR filter/decimator. As before, the effect of decimation can be controlled from the FIR Coefficient Editing dialog as shown below with the Ratio parameter (M). When the value is -2, a 2:1 decimator is produced. We can change the value to 1 to produce a normal FIR filter.

The Black curve below shows the output with decimation, while the Blue curve shows the FIR filter without decimation. This time the edge frequencies of the filter are 4.5kHz and 5kHz. Since the highest sampling rate involved with this stage is 20kHz, the system frequency range was changed to match.

The Black curve shows a periodicity of 10kHz, which is of course the output sampling rate of the decimator. This stage contains no aliasing and provides the final sharp transition from the 4.5kHz frequency to the Nyquist frequency of 5kHz.

The final clean output spectrum now lies in the range of 0Hz to 4.5kHz.

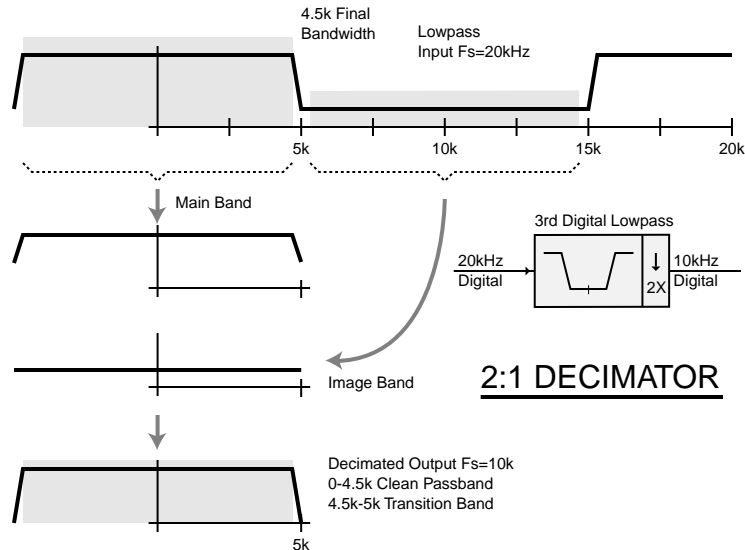


The output sampling frequency for this stage is 10kHz. In order to keep the desired bandwidth of 0Hz - 4.5kHz clean, the filter must remove all higher frequency regions which would fold back across the 4.5kHz band.

The diagram below shows how the aliasing occurs in this 2:1 decimator. The top graph shows the response of the 3rd FIR filter. The $\pm 4.5\text{kHz}$ gray boxes centered at multiples of 10kHz indicate the regions which would fold back on the final output spectrum. These areas must be fully attenuated.

For a 2:1 decimator there is only a single image band. The stopband edge begins at the Nyquist frequency (relative to the output sample rate), so the entire image band is fully attenuated.

This design produces a clean transition band. However in some designs aliasing is allowed to occur in the transition band.



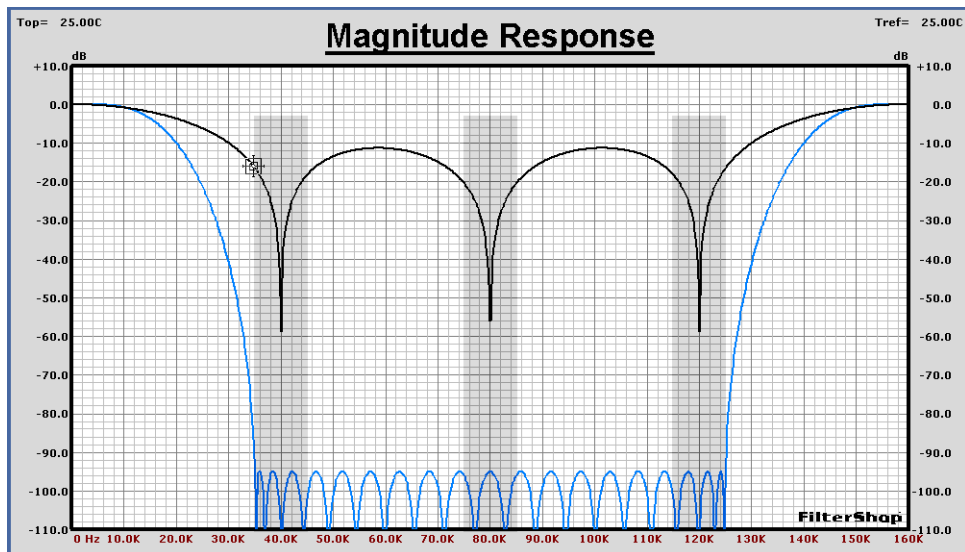
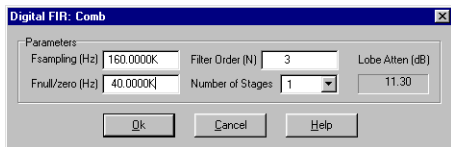
Using a Comb Filter as the 1st Stage FIR / Decimator

The previous discussions were used to illustrate where aliasing can occur in each of the decimation stages. However, the Multirate dialog performs all of the calculations automatically and creates the required filters for all stages.

We could certainly proceed with the design using these three FIR filters, but instead we will examine the possible use of a *Comb* filter in the first FIR stage. The results of the Multirate dialog are still very useful for obtaining the basic setup requirements of any other filters we may wish to utilize.

Comb filters are simple filters with identical coefficients of unity. As such they require no coefficient multiplication requiring only addition. Remember that the first stage in a decimator design also runs at the highest sampling rate. Simplifying the filter used in the first stage of a decimator can be very attractive.

Returning to the frequency spectrum of the first stage, note that the only regions which must be fully attenuated are narrow bands at the 40kHz multiples. This is shown below in the gray regions. If we apply a single 3rd order comb filter with a first null of 40kHz, the response shown below in the Black curve is produced.

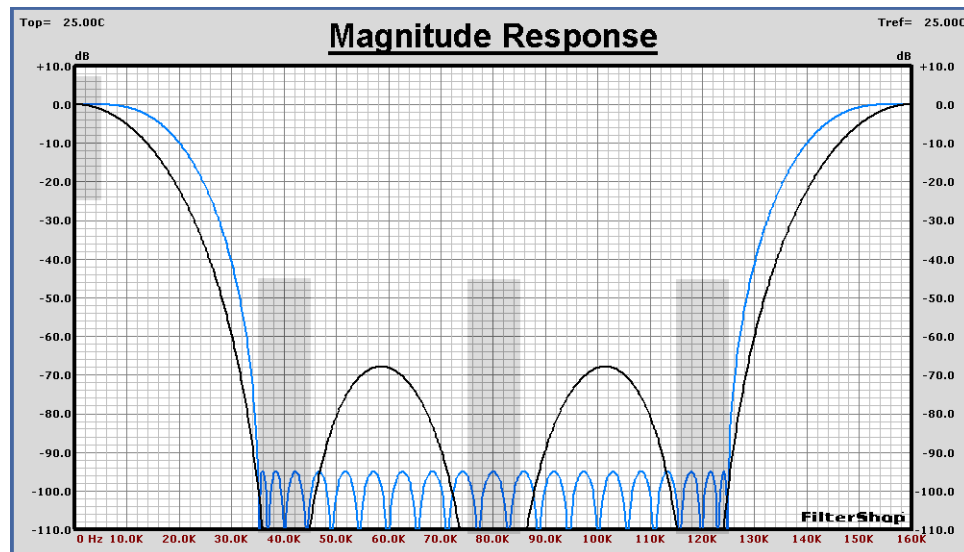


Each repetitive null of the comb filter lies exactly on top of all the regions which must be removed. However, the intersection of the comb filter response at the edges of the gray regions is only 15dB down. Clearly this does not meet the design objective.

If we now cascade several identical comb filter sections, the attenuation will be increased and the nulls widened. A couple of trial & error attempts quickly showed that six stages produce the required 100dB minimum. This six stage comb response is shown below in the Black curve.

In comparison to the original first stage response in Blue, the comb filter provides equal to or more attenuation. The chief difference is that the passband region (5kHz) is no longer flat. The response droops about -2dB at 5kHz. However this can be corrected in the design of the 3rd FIR stage.

The total order of the comb filter solution is 3×6 or 18. The original first stage design required an order of 24. However the most important difference is that the comb filter coefficients are all unity. This can be very important at high sampling frequencies or where custom filter hardware designs are used.



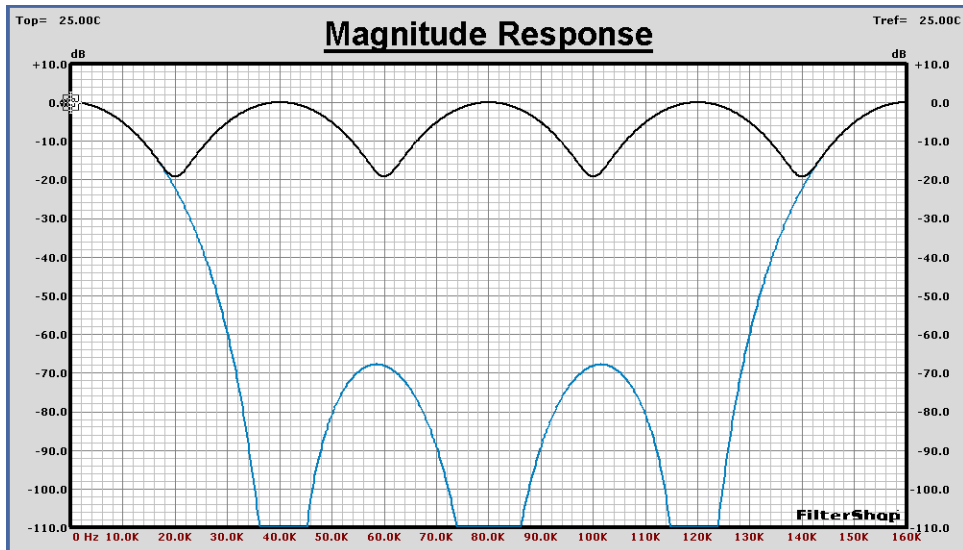
TFB	Mag	Phs	Eqn Gp	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Inv	Rev	M1	FIR	0.0000	160.0000K			3	H(0000) = 250.0000m		
2	Inv	Rev	M1	FIR	0.0000	160.0000K			3	H(0000) = 250.0000m		
3	Inv	Rev	M1	FIR	0.0000	160.0000K			3	H(0000) = 250.0000m		
4	Inv	Rev	M1	FIR	0.0000	160.0000K			3	H(0000) = 250.0000m		
5	Inv	Rev	M1	FIR	0.0000	160.0000K			3	H(0000) = 250.0000m		
6	Inv	Rev	M1	FIR	0.0000	160.0000K			3	H(0000) = 250.0000m		

The comb filter now consists of six TFBs in the Target Parameters as shown here. Each is a 3rd order FIR.

To simulate the multi-section comb filter as a decimator, we must convert the multiple FIRs into a single FIR. The decimator function is applied per TFB.

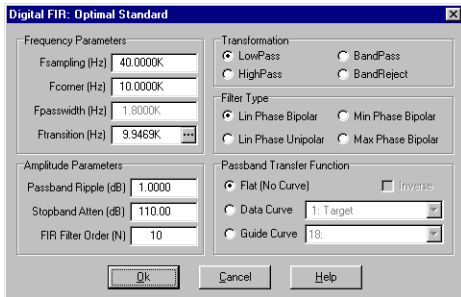
We can use the *Frequency Sampling* approximation method with an FIR of order 18. An exact equivalent of the six section comb filter is then produced as a single FIR TFB. The Ratio (M) parameter is then changed to -4 to represent a 4:1 decimator.

The graph below shows the final response of the comb filter with and without decimation. This completes the first FIR filter.



Reducing the 2nd Stage FIR / Decimator

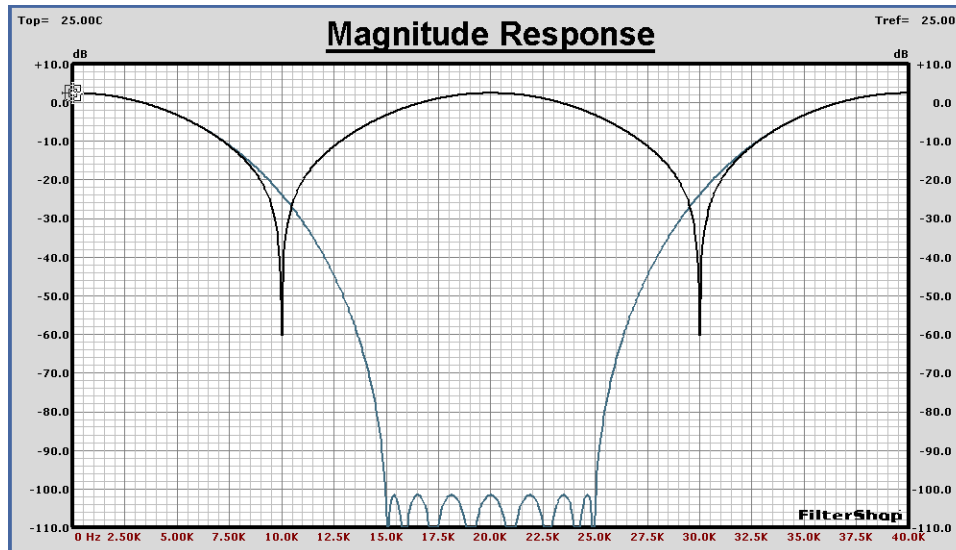
The original design from the Multirate dialog for the 2nd stage required an order of 16. This was based on the original passband ripple spec of 0.01dB being divided across all three of the stages.



Since we will be correcting the non-flat response (-2dB) of the comb filter, we might try designing this stage with much higher ripple. This should reduce the order. Using the Optimal Standard dialog, the parameters were entered along with an attenuation of 110dB and 1dB ripple as shown here on the left.

Low order filters have very coarse order estimation accuracy, so the 100dB desired attenuation was increased to 110dB to ensure the solution meets the minimum.

The resulting order is 10. This is almost 1/2 the original order for the second stage. The response with and without decimation for the second stage is shown below. The initial magnitude at DC came out at +2dB, and the attenuation did rise up from 110dB to 101dB. But this meets our goal.



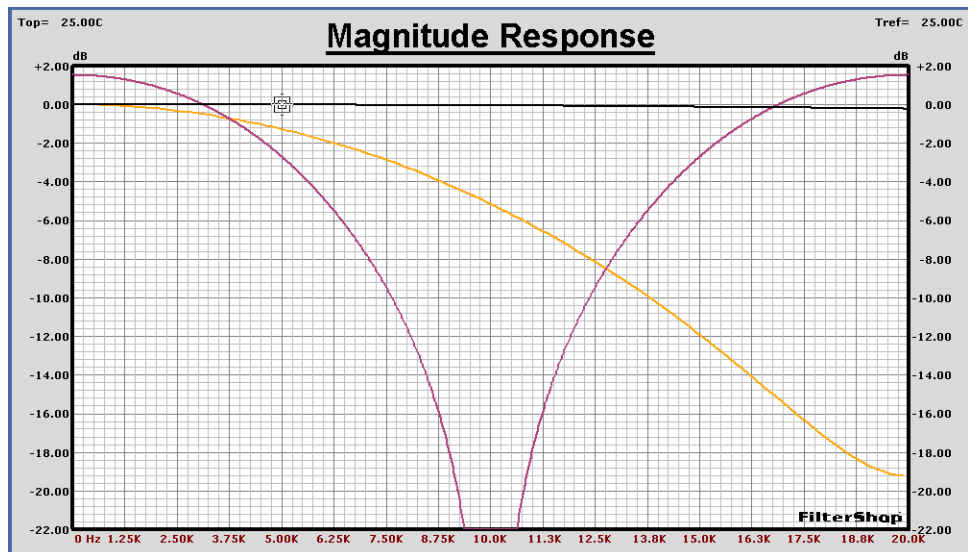
■ ADC Sin(x)/x Transfer Function

As mentioned previously we also wish to correct for a $\sin(x)/x$ (sinc) response based on a 160kHz frequency in the ADC. This can be trivially created in the Target Parameters using a single SINC filter type TFB with $F_s = 160\text{kHz}$. This is shown below in the Black curve.

While the sinc response appears nearly flat in the 5kHz passband, the response actually has a droop of almost -0.015dB at 5kHz. If we are to meet the 0.02dB passband flatness specification, this must be included in the design.

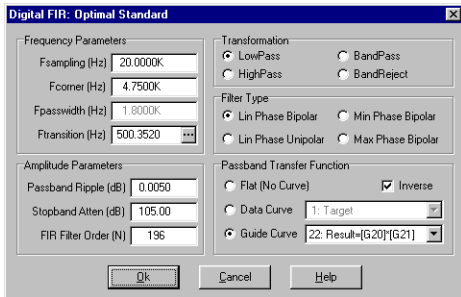
The other two curves show the response of the previous 1st and 2nd FIR/Decimator stages. The Orange curve is the 1st stage, and the Red curve the 2nd stage.

These three curves are multiplied together to form a single error response. This can then be used to provide correction in the final 3rd FIR/Decimator stage.



■ Correcting Responses in the 3rd Stage FIR / Decimator

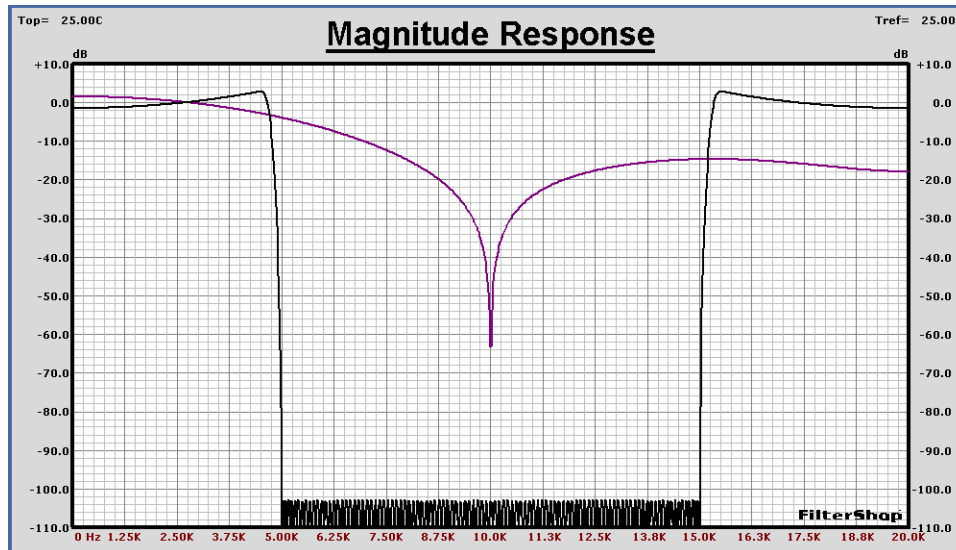
The high order 3rd stage FIR filter corrects all of the previous magnitude errors, in addition to providing the precision cutoff at the 4.5kHz corner. Here we again use the *Digital-FIR: Optimal Standard* dialog.

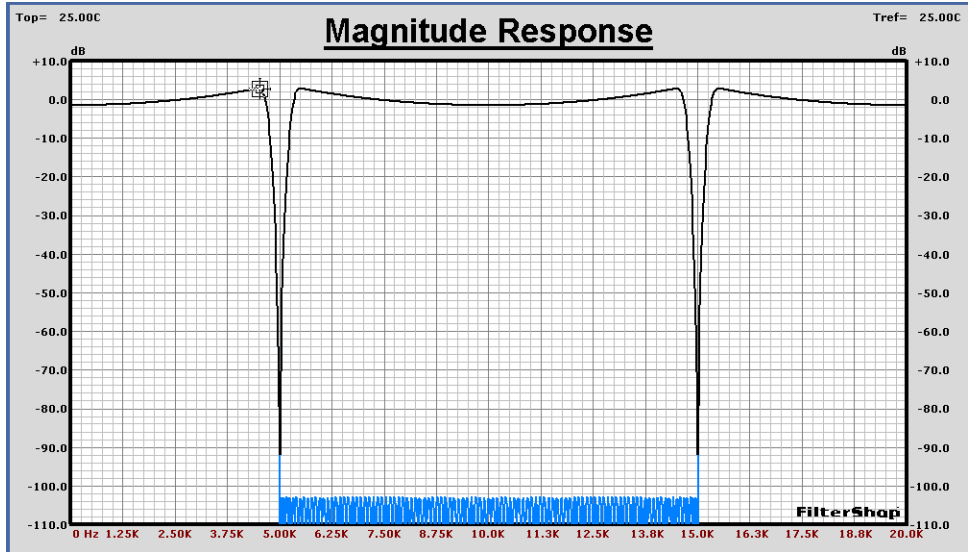


This filter uses $F_s = 20\text{kHz}$, with edge frequencies of 4.5kHz and 5kHz. We will slightly understate the ripple as 0.005dB and the attenuation as 105dB to ensure the solution meets the specifications.

The final step is to select the Guide Curve containing the combined error response and enable *Inverse*. The computed order is $N = 196$. The resulting FIR response is shown below, along with the error curve.

The complementary shape in the passband of the FIR filter is clearly visible. When the ADC/Sinc and three FIR/Decimators are combined, the total passband response should be flat with about 0.005dB ripple. The graph on the following page displays the response with and without decimation.



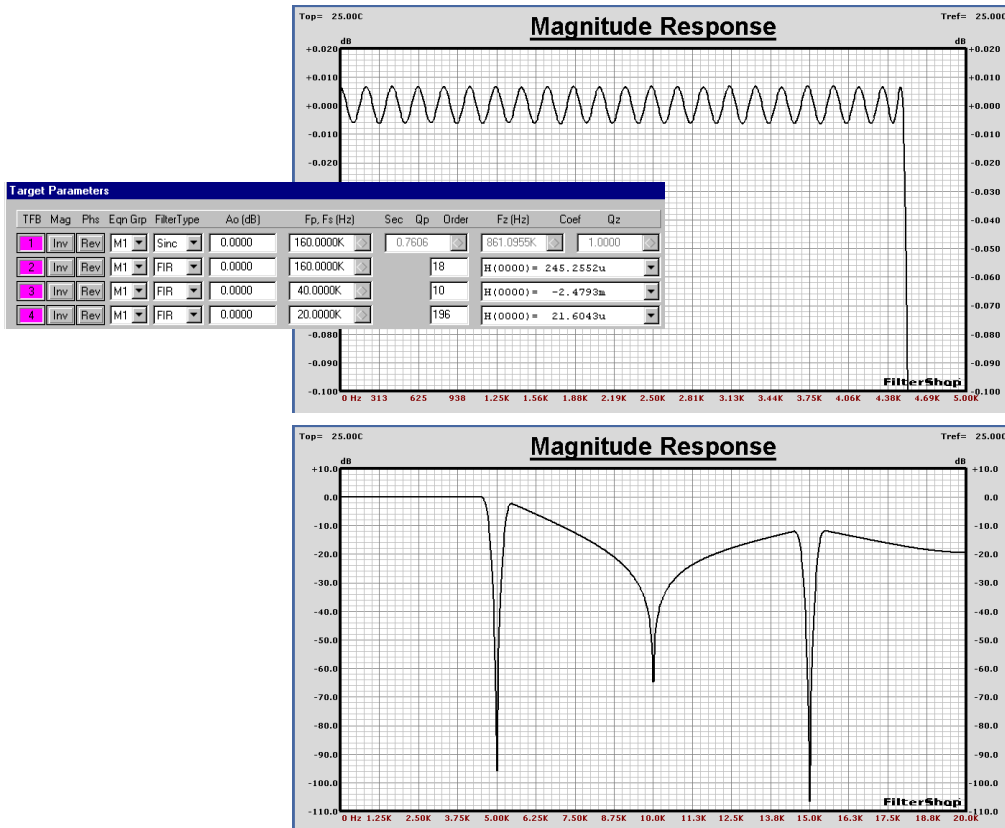


■ **Combined Response of ADC/Sinc and 3-Stage FIR/Decimator**

All of the previously designed blocks are now enabled in the Target Parameters as shown below. The two graphs show expanded views of the passband and stopband regions.

As expected the passband ripple is about 0.006dB. The lower graph shows an extended frequency range covering the higher stopband regions.

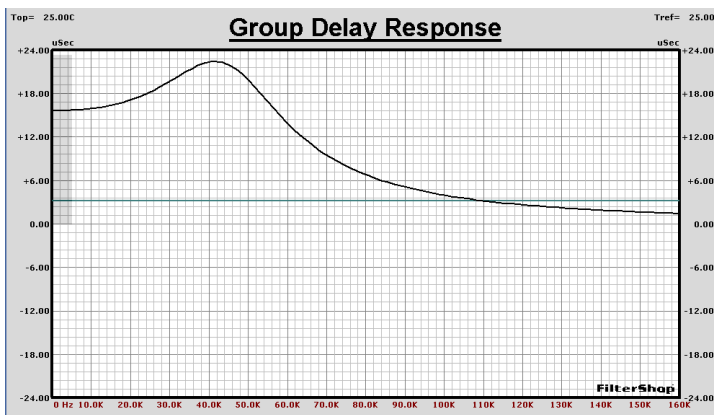
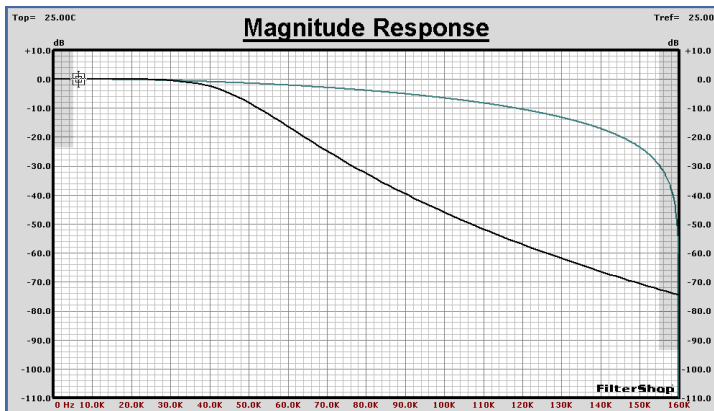
The response at frequencies above the Nyquist (5kHz) of the main output cannot be simulated accurately as a linear system, where all transfer functions are multiplied together. However, the data below the Nyquist is accurate and the higher frequency regions would be repeated images of this same data.



■ Analog Anti-Aliasing Filter Target

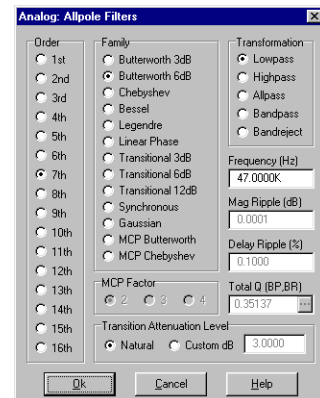
A filter must be placed before the ADC to prevent aliasing at the 160kHz sample rate. This of course must be an analog filter. The $\sin(x)/x$ loss of the ADC sampling provides about 30dB of attenuation at the lower 5kHz image edge at 155kHz as shown below. This leaves about 70dB of attenuation that must be provided by the analog filter.

One might first think that a flat group delay family would be required. However with a decimation ratio of 16:1 the transition bandwidth is wide enough that other filters can also be used. If the corner frequency of the filter is high enough, the group delay at low frequencies will be nearly flat for many filter families.



A Butterworth-6dB was chosen since this provides a compromise between the Bessel and standard Butterworth-3dB. A 7th order LP with $F_o=47\text{kHz}$ was chosen. The response is shown here in Black.

The group delay filter response in the gray region at 5kHz is virtually flat. The magnitude response in this same 5kHz range is less than 0.000005dB.



■ Analog Circuit Realization

To produce the 7th order analog Lowpass, a cascaded two stage design will be used consisting of 4th and 3rd order sections. The current Target Parameters for this filter are shown below.

Target Parameters													
TFB	Mag	Phs	Eqn	Gip	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Inv	Rev	M1		LP2	0.0000	47.0000K	1.3066		2	1.3317K	1.0000	
2	Inv	Rev	M1		LP2	0.0000	47.0000K	1.0000		2	1.6479M	1.0000	
3	Inv	Rev	M1		LP2	0.0000	47.0000K	0.5412		2	1.4317M	1.0000	
4	Inv	Rev	M1		LP1	0.0000	47.0000K	0.5412		1	1.5813M	1.0000	

There are three 2nd order TFBs and one 1st order TFB associated with the filter. We can choose to pair these up anyway we like to form the 4th and 3rd order targets. In order to produce a circuit with the best possible sensitivity, it is a good idea to pair low Q sections with higher Q sections.

The highest Q TFB is #1 so this will be paired with TFB #4 which is the 1st order TFB. This will form the 3rd order section. The 4th order section will be produced using TFB #2 and #3 with Q values of 1.0 and 0.5412. After a little rearranging the 4th and 3rd order sections are shown below.

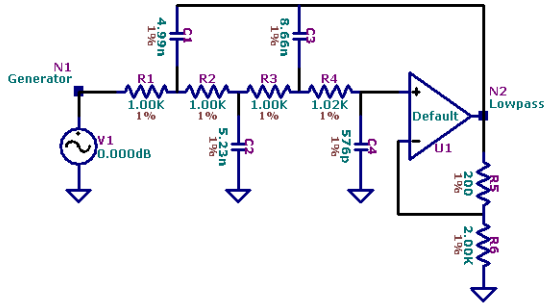
Target Parameters													
TFB	Mag	Phs	Eqn	Gip	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec	Qp	Order	Fz (Hz)	Coef	Qz
1	Inv	Rev	M1		LP2	0.0000	47.0000K	1.3066		2	1.3317K	1.0000	
2	Inv	Rev	M1		LP2	0.0000	47.0000K	1.0000		2	1.6479M	1.0000	
3	Inv	Rev	M1		LP2	0.0000	47.0000K	0.5412		2	1.4317M	1.0000	
4	Inv	Rev	M1		LP1	0.0000	47.0000K	0.5412		1	1.5813M	1.0000	
5	Inv	Rev	M1		LP2	0.0000	47.0000K	1.3066		2	1.3317K	1.0000	
6	Inv	Rev	M1		LP2	0.0000	47.0000K	1.0000		2	1.6479M	1.0000	
7	Inv	Rev	M1		LP1	0.0000	47.0000K	0.5412		1	1.5813M	1.0000	

Each of the two sections will be enabled/disabled alternatively to design each respective circuit section individually. After that they will be combined into the complete 7th order circuit.

■ Analog Filter Circuit: 4th Order Section

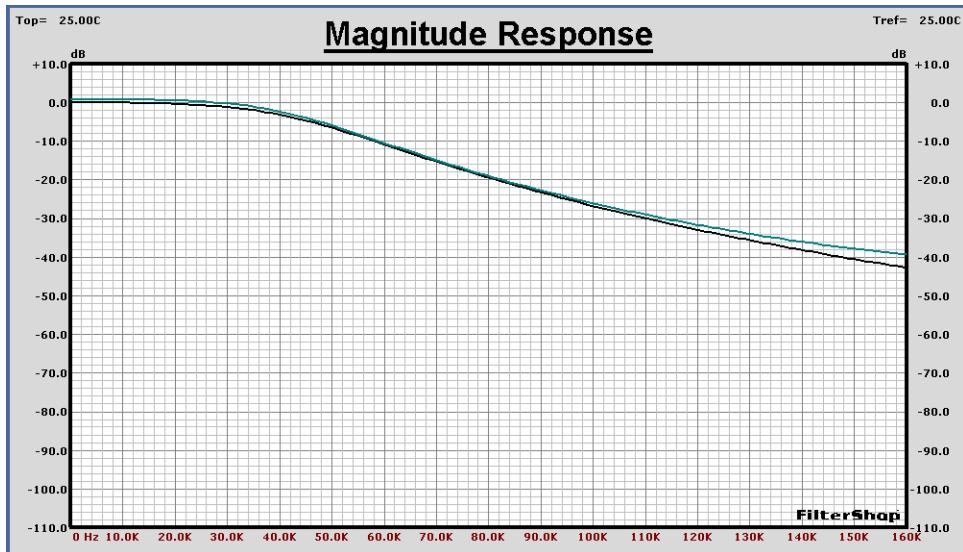
The LP4_SK1 synthesis circuit is loaded, and only TFBs #2 and #3 enabled. The 4th order Sallen-Key circuits can be tricky to design. Enough gain must be used to obtain a solution, but too much gain raises the component sensitivity. For this design the preset values of Gain = 1.1, 1K, and 2K were used for Synthesis. The circuit and its response is shown below as compared to the ideal target.

The circuit has a gain of 1.1 (0.83dB) where the target is at 0dB, and at high frequencies the curves begin to diverge. This is due to the limited bandwidth of the opamp. It will be important to simulate the circuit with a specific opamp.



To change the gain back to unity, R1 will be increased slightly and a resistor added to ground. We will also want to use 10% standard values for the capacitors. The opamp will be changed to a 5534 type.

The Target curve is copied to a Guide Curve, and then the revised circuit is optimized.

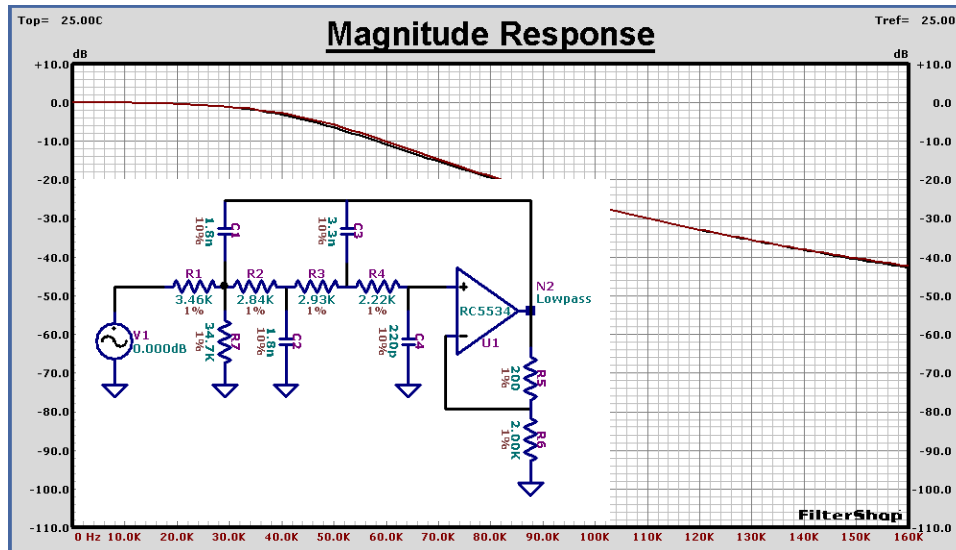
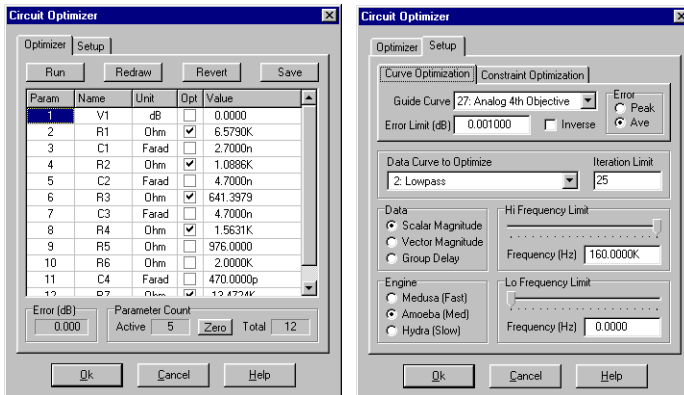


The optimization frequency range is initially the entire 160kHz bandwidth, with all components at 1% precision. Compensation for the opamp finite bandwidth will require changes in the capacitor values. In order to determine what the best range is for the capacitors, all of the components are optimized except for R5 and R6.

The result of the optimization is a very close match to the curve everywhere. The capacitors are then changed to 10% precision, disabled for optimization, and then the circuit reoptimized again using the resistors.

When this is completed, the frequency range is decreased to a 20kHz bandwidth to maximize the fit in the critical passband region.

The 5kHz region now matches the target to within 0.004dB. The final results are shown below.

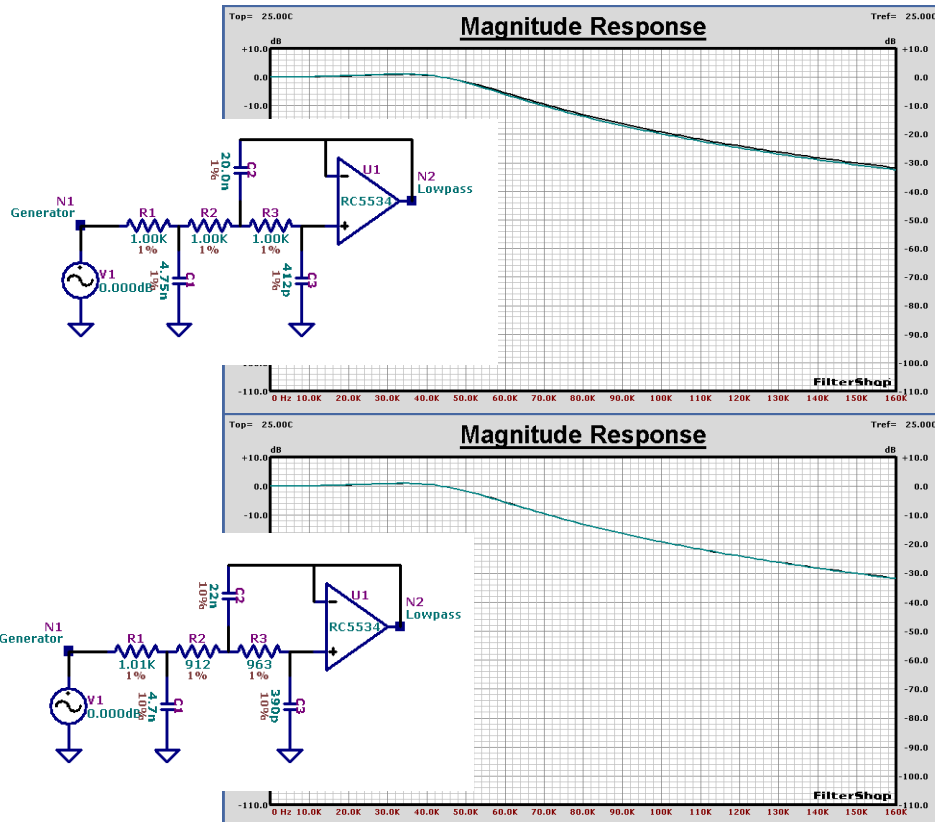


■ Analog Filter Circuit: 3rd Order Section

The LP3_SK1 synthesis circuit is loaded, and only TFBs #6 and #7 enabled. The synthesis for this circuit is straightforward. The synthesis preset resistor is set to 1k. The resulting circuit and response is shown below.

As with the previous circuit, the finite bandwidth of the opamp causes some difference in the response at high frequencies. This circuit will also be optimized.

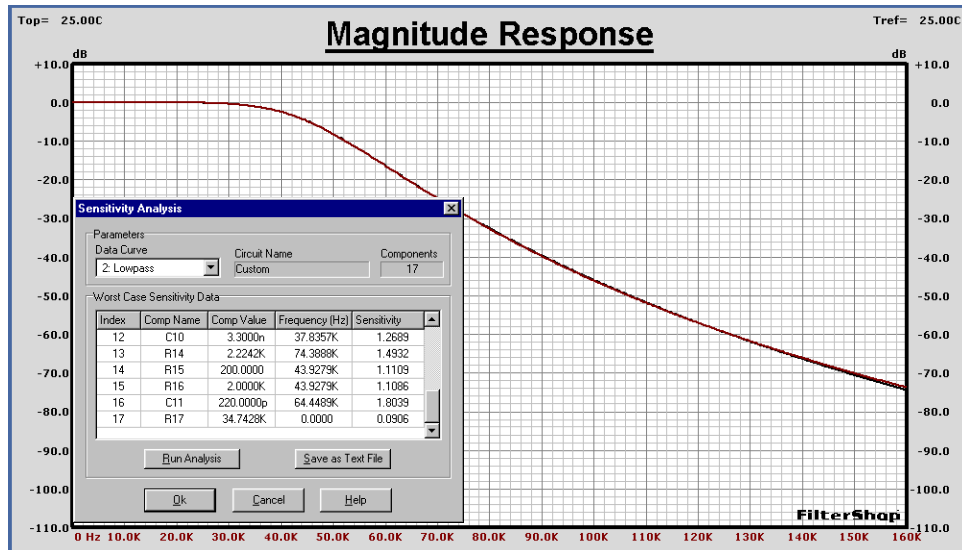
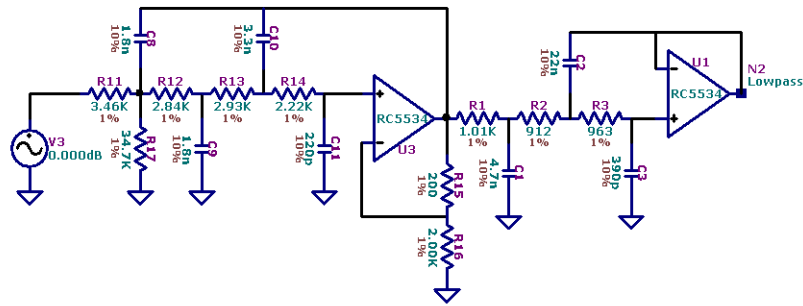
The capacitors are first changed to 10% standard values, and this time just the three resistors are optimized. After the circuit is optimized the response is virtually on top of the ideal 3rd order target everywhere.



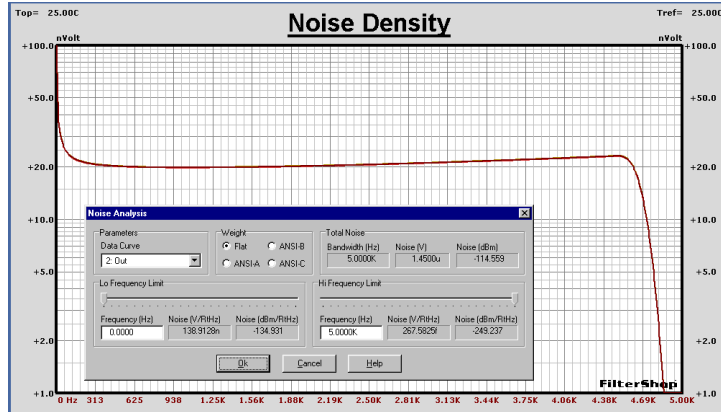
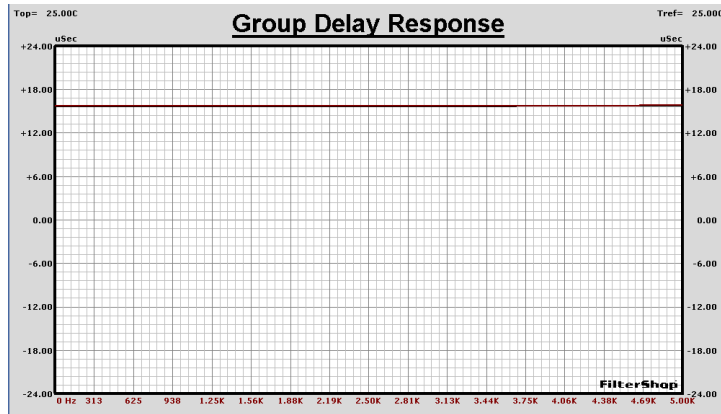
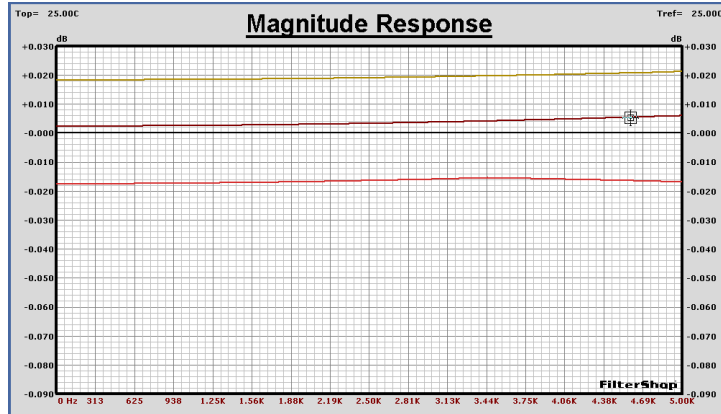
Complete Analog Filter Circuit

The 3rd and 4th order sections are now combined to form the complete analog filter. The circuit and its response is shown below. Sensitivity analysis shows excellent results with the worst case S value of 1.8 for C11. Most of the other values are below 1.5.

The detailed passband response is shown on the following page, along with a Monte Carlo analysis assuming 1% component tolerance. The Max/Min curves are between $\pm 0.02\text{dB}$ and the group delay is very flat. The broadband noise shows about 1.5uV (-114dBm) across the 5kHz bandwidth.



Complete Analog 7th Order Filter Passband Response

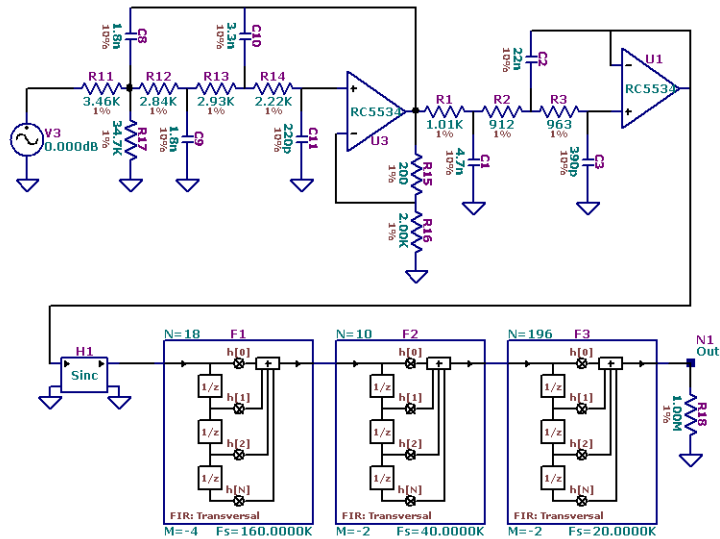


■ Complete Decimator Filter Simulation

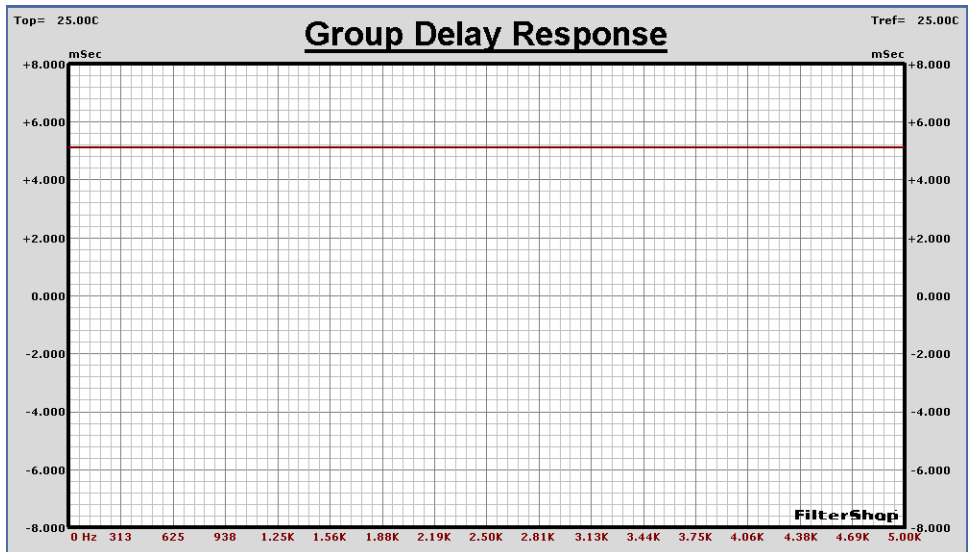
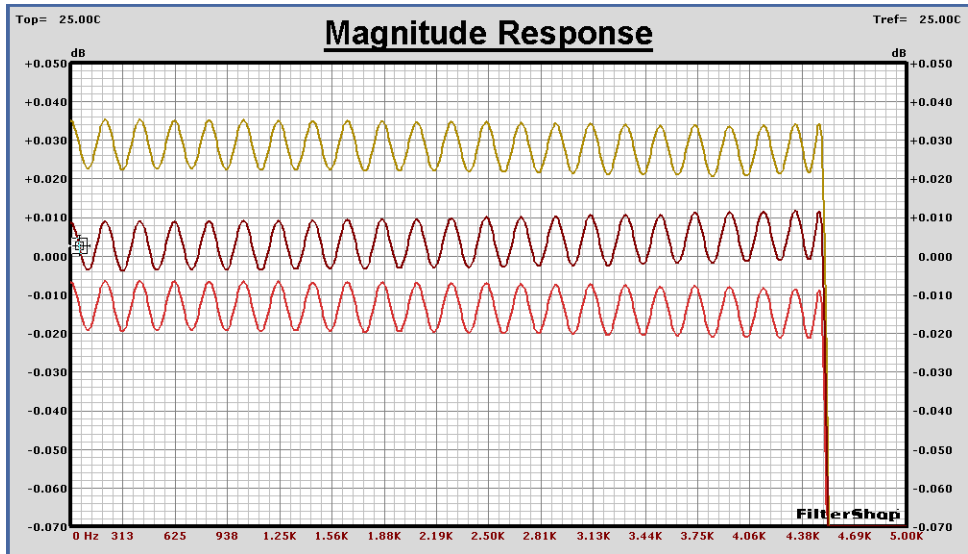
The entire analog/digital decimation filter design can now be verified. A circuit is created as shown below to provide the frequency domain simulation. The 7th order analog Lowpass filter follows the generator. An H transfer function component is used and setup as the Sinc function at 160kHz to represent the ADC. Three FIR components are used for each of the three decimation stages, and the target coefficient data is transferred into the components.

The two graphs on the following page display the results. The first graph shows the passband frequency range from 0Hz to 5kHz with 0.01dB/Div. Monte Carlo Max/Min response is also shown.

The average base gain is +0.005dB with a range of about ±0.025dB including ripple. The group delay response is very flat with a value of 5mS.



Complete Decimator Filter System Passband Response



■ Summary

The multirate FIR design dialog greatly simplifies the design of decimators and other multirate filters. The optimum order for each stage is provided along with the determination of the edge frequencies. This information allows for the easy design of different types of filters such as a cascaded comb FIR.

The correction of both digital and analog transfer functions can be performed with high accuracy and relative ease. The mixed domain filters can also be simulated together as a complete system.

This completes the Digital FIR Decimator & Analog Lowpass design.